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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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THE EVOLUTION OF BINARY STARS

By WILLIAM MARKOWITZ

ABSTRACT

I. The present theories on the evolution of binary stars are reviewed.

II. The effect of tidal forces on the period P of a binary is discussed for various stellar models. For the polytrope $n=3$, P can increase, at most, by a factor of about 1.4. The increase is practically zero for the models of Jeans and of Milne.

III. The densities of the components of eclipsing variables was found to be independent of the relative separation. The conclusion drawn is that the components are not contracting.

IV. A discussion of the distribution of eclipsing binaries with respect to the relative separation indicates that their absolute separations may be decreasing.

V. It is shown that a secular decrease of mass cannot convert the most massive spectroscopic binaries known into visual binaries of the average mass of the latter class. If the periods of spectroscopic binaries are being lengthened through a secular decrease of mass then there should be a statistical relation of the form $M=D_1/P^{\frac{1}{3}}$, where M is the total mass of the binary and D_1 is a constant. It is shown, however, that M is independent of P. The average value of M of a visual binary is not much smaller than the average value for a spectroscopic binary.

VI. It is doubtful if binaries whose periods are greater than about 55 days and less than several years have been formed from long-period binaries through the effects of close encounters with other stars.

VII. A resisting medium decreases P, a, and e of a binary. It is shown that these elements may decrease even if it is radiating away mass faster than it is picking it up.

VIII. Whether or not binaries can pick up enough interstellar matter to compensate for the loss by radiation cannot be decided definitely at present. Certain diffuse nebulae are probably dense enough to form O- and B-type stars.

The conclusion drawn from the data considered in this paper is that *if there are no forces acting on a binary which tend to increase P, a, and e other than those due to tides, secular decrease of mass and close encounters, then, in general, P, a, and e are decreasing.*

I. INTRODUCTION

The purpose of this investigation is to determine, if possible, whether spectroscopic binaries have evolved from visual binaries or vice versa. The following four hypotheses have been suggested to account for the formation of binary stars: (1) condensations around

separate nuclei in a nebula; (2) fission of stars; (3) action of a resisting medium; (4) growth of a planetary system. We shall assume that binaries have been formed in all of these ways, though there is no direct astronomical evidence that any given binary has been so formed.

The only forces known at present which can influence the period P, semi-major axis a, and eccentricity e of a binary are those due to (i) tides; (ii) perturbations due to other stellar systems (including close encounters); (iii) friction in a resisting medium; (iv) secular decrease of mass. We shall assume that these forces are, and have been, acting on binaries.

The first hypothesis on the formation of binaries will not be considered in detail, at this point, and we shall proceed to a discussion of the others, the fission theory first.

If the source of a star's energy is due only to its gravitational potential, then in the course of time it will contract. The moment of momentum of the star must remain constant, and the angular velocity, unless it were initially zero, would increase. The figures of equilibrium of rotating bodies have been the subject of extensive researches by several mathematicians, notably Colin Maclaurin, Karl Gustav Jacobi, Henri Poincaré, Sir George Darwin, Karl Schwarzschild, Alexander Liapounoff, and Sir James Jeans. Most of the work has been done on homogeneous, incompressible fluid models.

Maclaurin showed that for small values of $Q=\omega^2/2\pi k^2\rho$ (ω being the angular velocity, k^2 the gravitational constant, and ρ the density), the figure of stable equilibrium was an oblate spheroid of revolution whose oblateness vanished with Q. These stable spheroids, as was shown later, exist for all values of Q lying between 0 and 0.1871. Jacobi then found a series of stable ellipsoids of three unequal axes for $0.1420 \leqq Q \leqq 0.1871$. For $Q=0.1871$ the Jacobi ellipsoids and the Maclaurin spheroids coincide. Poincaré found a series of unsymmetrical figures of equilibrium, the so-called "pear-shaped," which exist for values of Q smaller than those giving the stable ellipsoids. Darwin showed that the ellipsoids and the pear shapes coincided when $Q=0.1420$.

Let us consider, then, the sequence of figures of stable equilibrium of a series of rotating homogeneous incompressible fluid bodies having the same mass and density, rotating as rigid bodies, but having

increasing moment of momentum. For zero moment of momentum (zero angular velocity) the figure is a sphere. With increasing moment of momentum, and ω, we have the spheroids of revolution. With still higher values of the moment of momentum, but with decreasing ω, the stability passes to the ellipsoids of Jacobi. These become more and more cigar-shaped as the moment of momentum increases and ω decreases until finally Poincaré's figures are reached.

It must not be supposed, as a consequence, that a contracting body will pass through the series of figures just mentioned, since ρ will increase with ω. But F. R. Moulton[1] has shown that such a body will become oblate as it contracts.

Let us consider the stability of Saturn. We shall assume that it is homogeneous, incompressible, etc. Saturn has a density of 0.72 (water = 1) and a rotation period of 10.25 hours, and for it $Q = 0.0983$. If an external force could be applied in such a manner as to reduce its rotation period to 7.43 hours, Q would equal 0.1871, and its figure would be one of the ellipsoids of Jacobi. From this it might be inferred that Saturn is not far from instability. But the actual case is quite different. If no external forces were applied, and increased rotation was due solely to contraction, then, as Moulton found, the figure of Saturn would not reach that just mentioned (for $Q = 0.1871$) until Saturn had a density of 21 and a period of rotation of 1.40 hours. Furthermore, the figure would not become pear-shaped ($Q = 0.1420$) until its density equaled 93 and its period 0.77 hours.[2]

The stability of the pear shapes was not determined by Poincaré. It was conjectured that, if they were stable, a furrow would be formed in the star as it kept on contracting, and that it would finally divide into two. If they were unstable then a star could never become pear-shaped, and a cataclysm, whose nature it is impossible to predict, would occur when the point was reached at which the Jacobi ellipsoids pass over to the pear shapes. Darwin put Poincaré's work in a numerical form and concluded that the pear-shaped figures were stable. According to Darwin, then, double stars were formed

[1] *Carnegie Institution of Washington Publications*, No. 107, p. 137, 1909.

[2] Eliminating ω and ρ from the expression for Q gives $Q = cV^2r/m \sin^2 i$, where c is a known constant, V is the equatorial velocity, r the radius, m the mass of the star, and i the angle between the axis of rotation of the star and the line of sight. The errors involved in the determination of V, r, and m (i is indeterminate) are so large that it is impossible to determine the stability of single stars from the foregoing point of view.

by fission, the two components revolving as a rigid body about each other and nearly in contact. The effect of further shrinkage of the components would be, through the agency of tidal forces, to increase P, a, and e.

Liapounoff, however, thought he had proved that the pear-shaped figures were unstable. Darwin and he were unable to come to an agreement on the matter, and the question of stability was left open until Jeans, in 1915, cleared up the discrepancy and showed that the figures were unstable. Nevertheless, Jeans, from a study of rotating liquid cylinders, a two-dimensional analogue of the problem here discussed, believes that short-period spectroscopic binaries have been formed by fission. Tidal forces being unable to account for more than a certain limited increase in P, a, and e, his views are that (a) binaries with periods greater than 55 days are due to separate condensations in a nebula, close encounters with other stars having served to reduce P and a; (b) binaries with periods less than 55 days have been formed by fission, tidal forces and a secular decrease of mass having increased P, a, and e; and encounters from passing stars being too rare to influence the orbits of such short-period systems.

If it is true, however, that the energy of a star is derived from the annihilation of matter, then, since a star that is radiating mass loses moment of momentum, the tendency toward fission will be eliminated unless, as Jeans believes, stars are subject to sudden contraction.

The third hypothesis concerning the formation of binary stars is a modification of the capture theory, but the latter term is somewhat misleading, and will be dropped. As long as there is no physical contact, two stars in space, not acted upon by exterior forces, cannot capture each other, no matter how close they approach. The effect of a resisting medium on a binary or on two independent stars, i.e., the orbital eccentricity, $e \geqq 1$, is to reduce P, a, and e.

For the formation of a double star by a resisting medium a very close approach is not required; an approach of two independent stars in a nebula to within 100 astronomical units might be sufficient to form a binary with a period of the order of 1000 years. From consideration of the fact that out of 15,000 known visual double stars only about 1000 show any traces at all of orbital motion, Hertzsprung[1] decided that the median period of the catalogued double

[1] *Bulletin of the Astronomical Institutes of the Netherlands*, 1, 149, 1922.

stars is probably thousands of years. Thus, on the basis of the friction theory binary stars would be formed with periods of several thousand years, the continued effect of the resisting medium serving to reduce P, a, and e.

The rarity of close approaches, coupled with the supposedly short life of a star, has hitherto made it seem improbable that this theory could account for the formation of a large number of binaries, especially those with short periods. But with the realization that the life of a star is much longer than previously supposed, and that it may even be indefinitely long, as will be pointed out in section viii, and in view of some of the factors mentioned in the preceding paragraph, it follows that this theory might be able to account satisfactorily for the large number of binaries known.

W. D. MacMillan has suggested that ultimately the solar system will become a binary. If the sun is losing mass by radiation, Jupiter, while picking up interstellar matter, is radiating but slightly. He believes that as the masses of Jupiter and the sun approach equality the other planets will probably fall into the sun or Jupiter, or suffer disruption from increased tidal strains. There is a possibility, of course, that a triple or multiple system may be formed. MacMillan has made a detailed study of the mathematical aspects of this hypothesis and of related cosmological questions.[1]

The problem of two bodies with diminishing mass has been discussed by Jeans,[2] MacMillan,[3] and E. W. Brown.[4] They find, on the assumption that the law of motion is Force = Mass × Acceleration, that the effect of a secular decrease of mass is to increase P and a, but to leave e invariant; P varying inversely as M^2, M being the sum of the masses, and a varying inversely as M. It will be shown in section vii that P, a, and e may decrease even if a binary is radiating mass faster than it is picking it up in its journey through space.

According to the fission theory a binary in which the components are nearly in contact is in the earliest stages of its existence, while on the basis of the friction theory it is in the last stages. Offhand, one

[1] *Astrophysical Journal*, **48**, 35, 1918; *American Mathematical Monthly*, **26**, 326, 1919; *Science*, **52**, 67, 1920; *Scientia*, **33**, 3 and 103, 1923; *Science*, **62**, 63, 96 and 121, 1925; *Monthly Notices of the Royal Astronomical Society*, **85**, 904, 1924–1925.

[2] *Monthly Notices of the Royal Astronomical Society*, **85**, 2, 912, 1924–1925.

[3] *Ibid.*, p. 904.

[4] *Proceedings of the National Academy of Sciences*, **11**, 274, 1925; **12**, 1, 1926.

might think that it would be easy to decide between such contradictory theories, but that is not the case. Practically everything that has been brought up as a confirmation of one of these theories could be shown to be equally consistent with the other. It is the purpose of this paper to investigate certain stellar statistics whose relations to theories of the evolution of binaries may be free from the objection just stated.

II. THE EFFECTS OF TIDAL FORCES

We have already mentioned Darwin's supposition that a binary, after having been formed by fission, would rotate as a rigid body; i.e., the orbital period and the rotation periods would all equal each other. He showed that further shrinkage of the components would, because of their increased angular velocity, set up a couple which would tend to increase P, a, and e.

Moulton,[1] however, by a consideration of the fact that the moment of momentum of the binary and its total energy (including the amount lost by tidal friction) must each remain constant, was able to show that tidal evolution, in so far as increasing P and a are concerned, is subject to very definite limits. He showed that in the case of binary stars where the masses of the components are comparable P could increase only by a small amount after fission.

In the case of binaries in which the components are comparable in mass, a consideration of the fact that the moment of momentum must remain constant is in itself sufficient to show how little tidal forces can increase P and a. The process of tidal evolution in this case is essentially a transference of the rotational moment of momentum of the components into the orbital moment of momentum. According to Darwin's theory the rotation periods of the components must always be equal to or less than the orbital period. In our discussion here we shall assume that the periods are always equal—the case most favorable to the fission theory. We shall also assume that the binary is not radiating any mass and that it is not acted upon by any forces which would tend to alter its moment of momentum.

Let A and B denote the two components, and let m_1 = the mass, a_1 = the distance from the center of A to the center of gravity of $A+B$, and r_1 = the radius of A. Similar symbols with the subscript 2

[1] *Op. cit.*, pp. 106, 153.

will refer to B. Further, let $M=m_1+m_2$, $a=a_1+a_2$, $\omega=2\pi/P=$ the angular velocity of the system $(A+B)$ about the orbital axis of revolution, and $I=$ the moment of inertia of the system about the same axis. The orbital eccentricity is small for close binaries, and is neglected. The value of the moment of momentum, which must remain constant, is

$$\omega I=\omega(m_1a_1^2+m_2a_2^2+\alpha m_1r_1^2+\alpha m_2r_2^2)\ . \tag{1}$$

The coefficient α depends upon the distribution of density within the star. If the star is homogeneous $\alpha=0.4$. The value of α was computed for the polytropic gas sphere $n=3$ from R. Emden's solution,[1] and found to equal 0.075. In this model, which Sir A. S. Eddington considers actually to be followed by the stars, the central density is 54.36 times the average density. For the liquid stars of Jeans and the models of E. A. Milne, which are highly condensed at the center, α is practically zero.

A study of the orbital elements indicates that the radii of the components are usually near equality. We shall assume, then, that r_1 equals r_2, and drop the subscripts on these letters. Expressing m_1, m_2 in terms of M and the mass ratio c; a_1, a_2 in terms of a and c, and eliminating ω from (1) by means of Kepler's equation,

$$\omega^2a^3=Mk^2\ ,$$

then, since the moment of momentum is constant, there results the equation

$$a=\frac{a_0(1+\beta)^2}{(1+\beta q^2)^2}\ , \tag{2}$$

where a_0 is the value of a when the components are in contact, and

$$\beta=\frac{\alpha(1+c)^2}{4c}\ , \qquad q=\frac{r}{a}\ . \tag{3}$$

The value of q when the components are in contact is evidently 0.5. The maximum value of a in equation (2) is obtained when q equals zero, i.e., when the components have shrunk to zero dimensions and have transferred all their rotational moment of momentum into

[1] *Gaskugeln*, Table 7, p. 80, 1907.

orbital. The value of the coefficient of a_0 in (2) has been computed for this case ($q=0$), for various values of α and c, and is entered in Table I.

The mass ratio, c, is rarely as small as 0.2, the average being about 0.75, and since the stars are probably as highly condensed toward

TABLE I

$c=$	1.00	.50	.20
$\alpha=.40$	1.96	2.10	2.96
.075	1.15	1.17	1.28
.00	1.00	1.00	1.00

the center as in Eddington's model or even more so, $\alpha \leqq 0.075$. Hence, tidal forces can increase a, at most by a factor of about 1.3, and P, by Kepler's law, by a factor of about 1.4.

In the fission theory eclipsing variables of the β Lyrae type, whose light-curves vary continuously, are supposed to be binaries only

TABLE II

Class	log 10P	P	β Lyrae	Algol	Sp. Binaries
I	.00	.100[d]	.....	.....	.....
II	.50	.316	8	1	.....
III	1.00	1.00	41	27	8
IV	1.50	3.16	16	153	62
V	2.00	10.0	9	130	86
VI	2.50	31.6	3	24	51
VII	3.00	100	3	6	28
VIII	3.50	316	1	4	27
IX	5.52	33,000	.....	2	41
Totals	.....	.....	81	346	303

recently formed. These are supposed to evolve into Algol-type variables and finally into spectroscopic binaries with periods up to about 55 days. In Table II, 81 β Lyrae variables, 346 Algol, and 303 spectroscopic systems have been divided into 9 groups according to period. Columns 2 and 3 contain the upper limit, respectively, of log 10P and P in each group. Columns 4, 5, and 6 give the number of systems of each class. The data for the eclipsing systems are taken

from R. Prager's *Katalog und Ephemeriden veränderlicher Sterne für 1931*,[1] and that for the spectroscopic binaries from A. Beer's *Zur Charakterisierung der spektroskopischen Doppelsterne*.[2]

More than 60 per cent of the β Lyrae stars have periods shorter than 1 day, while most of the Algol variables have periods between 2 and 10 days. On the other hand, there are almost as many spectroscopic binaries with periods greater than 10 days as there are with periods less than 10 days. Since tidal forces can increase P, at most, by three- or four-tenths of its value after fission, it follows that these forces cannot change the eclipsing variables into the ordinary spectroscopic binaries. In fact, from Table II it follows that tidal forces could not even change the β Lyrae variables into Algol variables. Since tidal forces decrease with the separation there should be no tendency for those members of a group which have the longer periods to evolve into the next group faster than those with the shorter periods.

III. THE DENSITIES OF ECLIPSING BINARIES

On the basis of the fission theory it is true that statistically the densities of the components A and B of a binary in which A and B are nearly in contact should be less than one in which the components are somewhat separated. Let the relative separation, σ, be defined as follows:

$$\sigma = \frac{\text{Distance between surfaces of } A \text{ and } B}{\text{Distance between centers of } A \text{ and } B}.$$

If A and B are in contact, $\sigma = 0$; while if A and B have shrunk to infinitesimal dimensions, $\sigma = 1$. There are two factors which can cause σ to increase: (1) an increase in the absolute separation (center to center); (2) contraction of A and B.

Let ρ_1 and ρ_2 be the densities of A and B, and $\rho_{1,0}$ and $\rho_{2,0}$ their values when $\sigma = 0$. Suppose that A and B contract but that the

[1] "Kleinere Veröffentlichungen der Universitäts Sternwarte zu Berlin-Babelsberg," No. 9, 1930.

[2] "Veröffentlichungen der Universitäts Sternwarte zu Berlin-Babelsberg," **5**, Heft 6, 1927.

absolute separation does not change. Then, assuming that the ratio of the radii of A and B does not change,

$$\rho_i = \frac{\rho_{i,o}}{(1-\sigma)^3} \qquad (i = 1 \text{ or } 2)\,. \tag{4}$$

Suppose, now, that tidal forces do increase the absolute separation and by the maximum amount possible. Then (2) must be satisfied. Eliminating a from (2) by means of the second of (3) and q by means of the relation,

$$\sigma = \frac{(a-2r)}{a} = 1-2q\,,$$

it is found that the expressions for the densities can be brought to the form

$$\rho_i = \frac{\rho_{i,o}[1+\beta(1-\sigma)^2]^6}{(1-\sigma)^3(1+\beta)^6}\,. \tag{5}$$

It is seen that when $\alpha = 0$, $\beta = 0$ and (5) reduces to (4).

If the mass ratio of an eclipsing binary is known, ρ_1 and ρ_2 can be computed from the orbital elements. The densities and relative separation of the components of 125 eclipsing binaries have been plotted in Figure 1. Of these 125, 90 are from H. Shapley's catalogue of eclipsing binaries.[1] The data for 14 of Shapley's list for which more recent orbits have been computed, and for 35 others not in his list, are given in Table III. Column 3 gives σ. The densities of the components ($\odot = 1$) are given in columns 4 and 5. The sixth column gives the ratio of the densities, p; it is always taken to be less than unity. The number in Shapley's catalogue is given in the first column.

The mass ratio was determined either from spectroscopic observations (when double-lined spectra are available) or from differences in brightness, or assumed to be unity. A small error in the mass ratio has little effect on the computed densities.

The average of the mean densities of 250 stars (125 binaries) was found to be $0.219 \times \odot$. The binaries were divided into classes, in Table IV, according to relative separation. Column 2 gives the upper

[1] *Contributions from the Princeton University Observatory*, No. 3, 1915.

limit of σ for the binaries in each class. Column 3 gives the mean density; 4, the number; 5, the average of the ratios of the densities; and 6, the number of binaries in each group for which $p < .10$. Four systems have $\sigma > .8$ and are not included in Table IV.

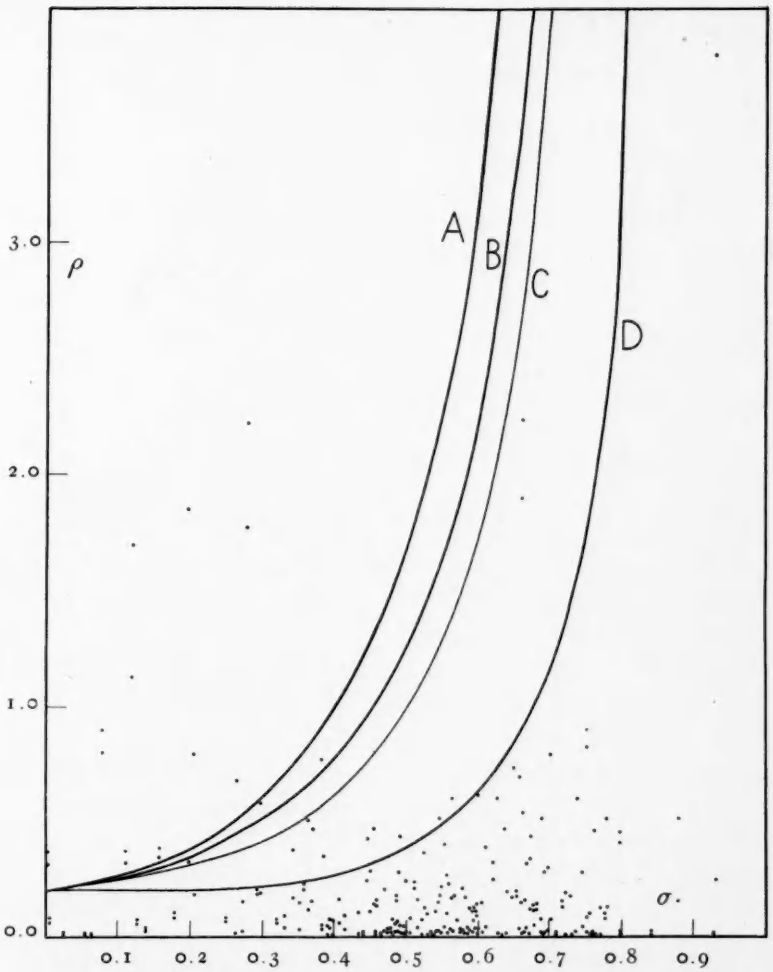

FIG. 1.—The densities of the components (sun = 1) of 125 eclipsing binaries plotted against their relative separations. If the components were contracting, then the points should cluster around one of the above curves; which one, depends on the effectiveness of tidal forces. The values of ρ for large values of σ are not indicated on the diagram. For $\sigma = 0.9$ the ordinates of the various curves are as follows: *A*, 210; *B*, 136; *C*, 97.5; *D*, 27.8.

In order to compare the observed densities with those to be computed by (4) and (5) it is necessary to determine ρ_0. From Table IV it is seen that the average value of ρ for class I, which includes the systems in which A and B are in contact or nearly so, is 0.21. This

TABLE III

THE DENSITIES AND RELATIVE SEPARATION OF THE COMPONENTS OF 49 ECLIPSING BINARIES

	Name	$\sigma = 1 - \frac{r_1 + r_2}{a}$	ρ_1	ρ_2	$\bar{\rho}$	Reference
	RV Ophiuchi	.64	.24	.06	.25	R. S. Dugan, *PO*, No. 4, 1916
	RZ Cassiopeiae	46	.45	.12	.27	*Ibid.*
5	U Cephei	.48	.21	.02	.10	*Ibid.*, No. 5, 1920
26	Y Camelopardi	.52	.06	.02	.33	*Ibid.*, No. 6, 1924
55	SZ Herculis	.33	.37	.17	.46	*Ibid.*
70	RS Vulpeculae	.53	.06	.008	.13	*Ibid.*
25	R Canis Majoris	.50	.51	.19	.37	*Ibid.*
	RY Aquarii	.57	.40	.06	.15	*Ibid.*
3	TV Cassiopeiae	.41	.13	.05	.39	*Ibid.*, No. 7, 1924
	TW Cassiopeiae	.67	.19	.18	.95	*Ibid.*
	TX Cassiopeiae	.14	.02	.007	.33	*Ibid.*
	T Leonis Minoris	.55	.11	.02	.18	*Ibid.*
	SS Camelopardi	.40	.08	.005	.06	*Ibid.*
	X Trianguli	.37	.47	.14	.30	*Ibid.*, No. 8, 1928
4	RT Sculptoris	.20	1.89	.35	.19	*Ibid.*
	WW Aurigae	.66	.21	.20	.95	*Ibid.*, No. 10, 1930
	W Ursae Minoris	.27	.05	.05	1.0	*Ibid.*
	RS Canum Venaticorum	.62	.45	.012	.03	B. W. Sitterly, *ibid.*, No. 11, 1930
13	λ Tauri	.48	.025	.024	.96	J. Stebbins, *AJ*, **51**, 193, 1920
11	β Persei	.55	.15	.03	.20	*Ibid.*, **53**, 105, 1921
	1H Cassiopeiae	.74	.28	.04	.15	*Ibid.*, **54**, 84, 1921
49	δ Librae	.39	.08	.03	.38	J. Stebbins, *WO*, **15**, 1928
	α Coronae Borealis	.934	3.8	.25	.07	*Ibid.*
	9 Andromedae	.53	.15	.04	.27	C. M. Huffer, *ibid.*
	21 Cassopeiae	.75	.51	.12	.23	*Ibid.*
	H.D. 25833	.55	.25	.10	.40	C. M. Huffer, *unpublished*
	Boss 1607	.87	.50	.14	.36	*Ibid.*
	SX Hydrae	.60	.62	.02	.03	M. B. Shapley, *HB*, No. 797, 1924
	V Leporis	.18	.10	.08	.80	*Ibid.*, No. 842, 1927
	FP Carinae	.927	.016	.007	.44	*Ibid.*, No. 843, 1927
	ZZ Sagittarii	.51	.08	.02	.25	*Ibid.*, No. 847, 1927
	TT Hydrae	.69	.52	.005	.01	*Ibid.*
	UW Virginis	.56	.49	.05	.11	*Ibid.*, No. 848, 1927
	S Equulei	.71	.39	.07	.18	*Ibid.*, No. 874, 1930
	TW Andromedae	.59	.12	.02	.17	M. B. Shapley, *AJ*, **56**, 439, 1922
	H.V. 3622	.73	.09	.02	.22	I. E. Woods and M. B. Shapley *HC*, No. 238, 1922
1	SX Cassiopeiae	.75	.09	.0005	.005	B. P. Gerasimovic, *HB*, No. 852, 1927
	SW Lacertae	.12	1.54	1.23	.80	J. S. Schilt, *BAN*, **2**, 175, 1924
	YY Geminorum	.65	2.2	1.9	.86	H. van Gent, *ibid.*, **6**, 99, 1931
	H.D. 1337	.02	.009	.003	.33	J. A. Pearce, *DO*, **3**, 275, 1927
	TU Herculis	.55	.40	.03	.08	J. Q. Stewart, *AJ*, **42**, 315, 1915
	SX Draconis	.49	.05	.007	.14	W. van B. Roberts, *ibid.*, p. 312, 1915
	RR Vulpeculae	.70	.30	.03	.10	M. Maggini, *ibid.*, **50**, 141, 1919
87	RT Lacertae	.45	.013	.013	1.0	M. Fowler, *ibid.*, **52**, 257, 1920
	σ Aquilae	.53	.15	.12	.83	C. C. Wylie, *ibid.*, **56**, 232, 1922
	CG Cygni	.34	.75	.35	.47	Ch'ing-Sung Yü, *ibid.*, **58**, 75, 1923
	S Antliae	.11	.38	.31	.82	A. H. Joy, *ibid.*, **64**, 287, 1926
61	RX Herculis	.62	.31	.30	.97	R. F. Sanford, *ibid.*, **68**, 51, 1928
19	TT Aurigae	.30	.12	.11	.91	Joy and Sitterly, *ibid.*, **73**, 77, 1931

ABBREVIATIONS: *AJ = Astrophysical Journal; BAN = Bulletin of the Astronomical Institutes of the Netherlands; DO = Publications of the Dominion Astrophysical Observatory; HB = Harvard Bulletin; HC = Harvard Circular; PO = Contributions from the Princeton University Observatory; WO = Publications of the Washburn Observatory.*

value is near the average value for all the binaries and we shall put ρ_0 equal to it. Graphs of equations (4) and (5) have been drawn in Figure 1, using various values of a and c. The curves drawn are

A. (4); or (5), $a=0.0$,
B. (5), $c=1$, $a=.075$,
C. (5), $c=.2$, $a=.075$,
D. (5), $c=1$, $a=0.40$.

If no tidal action takes place or if $a=0$ (the latter case implies the former), the points in Figure 1 should cluster around curve A. If the maximum amount of tidal action does take place, then, since the

TABLE IV

Class	$\sigma=1-\frac{r_1+r_2}{a}$	$\bar{\rho}$	n	p	u
I	.1	.211	6	.61	1
II	.2	.422	7	.82	0
III	.3	.520	8	.51	1
IV	.4	.167	11	.42	2
V	.5	.137	21	.45	1
VI	.6	.112	29	.22	4
VII	.7	.254	22	.30	10
VIII	.8	.197	17	.28	9

mass ratio in binaries is about 0.75, the points should cluster around D, if the stars are homogeneous, and around B if their law of density is that given by Emden's polytrope, $n=3$. Since it is obvious that none of these occurs, the conclusion drawn is that *the components of close binaries are not contracting.*

The results of this and the preceding sections show that tidal effects are negligible in the evolution of binary stars in which the masses of the components are comparable.

IV. FREQUENCY OF ECLIPSING BINARIES

Column 4 of Table IV shows the number of eclipsing variables in the various σ-classes. Contrary to expectation, most of these systems are not found in the classes having small values of σ (small separation between the surfaces). Curve A in Figure 2 is a smoothed

frequency-curve showing the number of systems to be found for an interval in σ of 0.1.

Let the two components of an eclipsing binary be equal in size and in luminosity, and let us assume that an eclipse will be visible to the

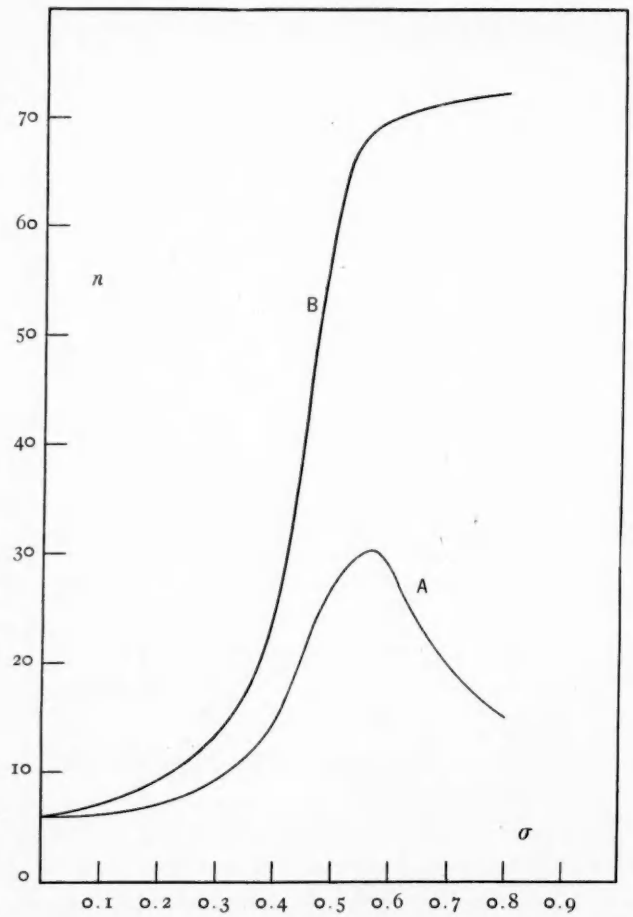

FIG. 2.—The distribution with respect to the relative separation of the eclipsing systems plotted in Figure 1 is given by curve A. The ordinates of B are proportional to the actual distribution.

earth if, at time of minimum, the portion of the radius of each star which is hidden $\geqq l$, l being a constant. The orbital eccentricities of eclipsing binaries are small and will be neglected. Let i be the com-

plement of the angle between the line of sight and the normal to the plane of the orbit. Let j be the maximum value of i for which an eclipse will be visible.

From Figure 3 it is seen that

$$\sin j = \frac{r(2-l)}{a} = \frac{(1-\sigma)(2-l)}{2}.$$

The chance that $i \leqq j$ is the same as the ratio of the area of the zone of a sphere lying between latitudes $+j$ and $-j$ is to the area of the sphere. Denoting this ratio by f, we have

$$f = 4\pi \int_0^j \frac{\cos\theta d\theta}{4\pi} = \frac{(1-\sigma)(2-l)}{2}.$$

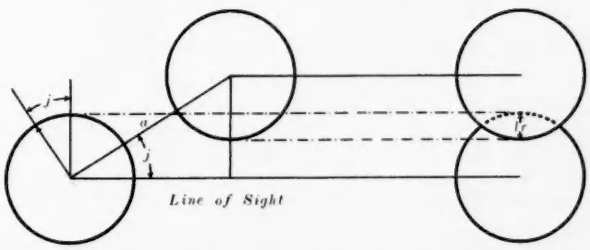

FIG. 3.—The configuration of the binary at the time of maximum eclipse

Hence, under the assumptions made, the chance that a given spectroscopic binary will be detected as an eclipsing system is proportional to $(1-\sigma)$. In order, then, to find the true distribution with respect to the relative separation, it is necessary to multiply all the ordinates of curve A in Figure 2 by $1/(1-\sigma)$. The resulting curve is labeled B. There is seen to be a sudden drop in the frequency as σ becomes less than .5.

Darwin, in a paper entitled "On the Figure and Stability of a Liquid Satellite,"[1] discussed the question of how close two equal liquid homogeneous incompressible masses could be brought to each other before tidal strains disrupted the bodies. From his results it is found that this happens when σ reaches a value of about 0.32. Stars actually found in nature increase in density toward the center,

[1] *Philosophical Transactions of the Royal Society*, A, **206**, 161, 1906.

and as this factor leads to greater stability, two equal stars can probably approach each other more closely than when $\sigma = 0.32$. On the other hand, if one star is much denser than the other, the latter may break up before σ gets as low as 0.32.

A glance at Figure 1 shows that for small values of σ the densities of the components of a given star are nearly equal (the two components have the same abscissa). Column 5 of Table IV gives the average value of p, the ratios of the densities, for each group, and 6 gives u, the number of binaries in each group for which $p < .10$.

There is an abrupt change in p and u as σ passes from .6 to .5. Hence, the conclusion that might be inferred from a study of Figure 2 and Table IV is that the components of close binaries are being drawn together, systems in which one component is much denser than the other being disrupted first.

V. SECULAR DECREASE OF MASS; VISUAL BINARIES

We have already stated that a secular decrease of mass will increase the orbital period, P of a binary; P varying inversely as the square of M, the sum of the masses. In Table V, P and M (present values) are tabulated for 22 short-period spectroscopic binaries whose inclinations are known. The data are taken from Table 45 of Beer's catalogue.[1] The masses that these systems would have after losing enough mass to become visual binaries with periods of 40 and 100 years, respectively, have been computed and entered in columns 5 and 6. The formulae used in their computation are

$$M(i) = M\left(\frac{P}{365.25i}\right)^{\frac{1}{3}} \qquad (i = 40 \text{ or } 100)\,.$$

R. G. Aitken[2] finds that the mean M of 14 visual binaries whose periods are of the order of 40 years is $1.76 \times \odot$. It is seen, then, that these spectroscopic binaries, which include some of the most massive known, could hardly hope to become long-period visual binaries of the type now known, through a loss of mass.

SPECTROSCOPIC BINARIES

Tidal forces being incompetent to increase P and a, and close encounters from passing stars being admittedly a negligible factor in

[1] *Op. cit.* [2] *The Binary Stars*, p. 209, 1918.

the evolution of close spectroscopic binaries, it remains for a secular decrease of mass to account for the increase in P and a after fission. If, indeed, P has been increased because of a loss of mass, then the following relation should be satisfied statistically, since, for any given binary, P varies inversely as M^2:

$$\overline{M}\overline{P}^{\frac{1}{2}} = D_1 , \tag{6}$$

D_1 being a constant. In applying this formula the range in P must be small for each separate group.

TABLE V

No. in Beer's Catalogue	Name	P	M	$M_{(40)}$	$M_{(100)}$
6	H.D. 1337	3.523^d	70.1$\times\odot$	1.090$\times\odot$	.689$\times\odot$
22	V Puppis	1.454	35.6	.355	.231
23	Y Cygni	2.996	31.9	.451	.286
35	u Herculis	2.051	10.3	.122	.077
41	Z Vulpeculae	2.455	7.6	.099	.062
49	U Coronae Borealis	3.452	5.9	.090	.057
57	U Ophiuchi	1.677	10.0	.107	.067
61	β Lyrae	12.92	23.4	.695	.440
75	RS Vulpeculae	4.477	5.7	.100	.063
76	σ Aquilae	1.950	11.3	.158	.083
78	β Persei	2.867	5.7	.079	.051
89	RX Herculis	1.779	1.9	.021	.013
95	β Aurigae	3.960	4.7	.078	.049
99	TV Cassiopeiae	1.813	2.7	.031	.019
129	ζ^1 Ursae Majoris	20.54	7.5	.283	.179
134	TX Herculis	2.060	3.9	.046	.029
160	WW Aurigae	2.525	4.1	.054	.034
164	S Antliae	.648	1.2	.008	.005
180	Z Herculis	3.993	2.9	.048	.031
184	RS Canum Venaticorum	4.798	2.5	.046	.029
218	W Ursae Majoris	.334	1.2	.006	.004
279a	YY Geminorum	.814	1.2	.009	.006

For spectroscopic binaries we have the following well-known relation

$$KP^{\frac{1}{2}} = \frac{cC_0 M^{\frac{1}{2}} \sin i}{(1+c)(1-e^2)^{\frac{1}{2}}} , \tag{7}$$

where K is the semi-amplitude of the velocity of the more massive component, C_0 is a constant, and c is the mass ratio. In a statistical study we may treat i, c, and e as constants. Hence, equation (7) becomes

$$\overline{K}\overline{P}^{\frac{1}{2}} = \overline{M}^{\frac{1}{2}} C_1 , \tag{8}$$

where C_1 is a constant. If (6) is satisfied, then (8) must become

$$\bar{K}\bar{P}^{\frac{1}{3}} = D\,, \tag{9}$$

where D is a constant.

O. Struve[1] has grouped 116 spectroscopic binaries into 9 period groups whose mean periods ranged from 2.43 to 3199 days, and he found that

$$\bar{K}\bar{P}^{\frac{1}{3}} = C\,, \tag{10}$$

C being a constant independent of the period group and having a value of 116.7. This implies, from (8), that M is independent of the period classification.

TABLE VI

THE PERIOD-AMPLITUDE FUNCTIONS

Class	Group	log 10P	P	$\bar{P}$	$\bar{K}$	n	$C=\bar{K}\bar{P}^{\frac{1}{3}}$	$D=\bar{K}\bar{P}^{\frac{1}{3}}$
I.....	1	.30	.1995[d]	.....	.....	.....	.....	.....
	2	.60	.3981	.347	143.7	3	101.0	85.7
	3	.90	.7943	.693	81.6	3	72.2	68.0
II.....	4	1.20	1.585	1.267	90.0	18	97.5	102.5
	5	1.50	3.162	2.362	84.4	45	112.3	129.7
	6	1.80	6.310	4.391	61.4	55	100.3	128.3
	7	2.10	12.59	9.430	52.2	48	110.3	160.2
	8	2.40	25.12	17.77	51.4	28	134.4	216.0
	9	2.70	50.12	34.71	36.2	20	118.0	213.0
	10	3.00	100.0	67.15	29.5	15	120.0	241.7
III.....	11	3.30	199.5	138.3	32.2	19	166.8	379
	12	3.60	398.1	260.9	18.7	9	119.2	302
	13	3.90	794.3	561.0	12.0	11	99.2	284
	14	4.20	1585	1027	15.1	10	152.5	484
	15	4.50	3162	2423	10.1	7	136.3	496
	16	4.80	6310	4111	14.4	5	231.0	924
	17	5.10	12,590	9350	7.25	4	152.8	700
	18	5.40	25,120	18,260	3.0	1	79.1	405
	19	5.70	50,120	30,840	2.0	2	62.7	350

In Beer's catalogue[2] 303 stars are available for discussion. They have been divided into 19 groups according to period as listed in columns 3 and 4 of Table VI. The interval in log 10P is taken to be 0.3. The values of P and log 10P are the upper limits for the systems

[1] *Astrophysical Journal*, **60**, 167, 1924. [2] *Op. cit.*

in each group. The 19 groups have been combined into 3 classes as indicated in column 1. In the other columns we have

2. the group number ,
5. $\overline{P}$,
6. $\overline{K}$,
7. n, the number in the group ,
8. $C=\overline{K}\overline{P}^{\frac{1}{3}}$,
9. $D=\overline{K}\overline{P}^{\frac{1}{2}}$.

Let C_i and D_i denote the values of C and D for each of the 19 groups. It is evident that there are not sufficient systems in the groups of classes I and III to give accurate individual values of C_i and D_i, but the results are tabulated for the sake of completeness. The weighted mean of the C_i, calling the number of stars in each group the weight, is 117.3. For the weighted means of the 3 classes we have

$$C \quad (\mathrm{I}) = 86.6 ,$$
$$C \quad (\mathrm{II}) = 111.1 ,$$
$$C \ (\mathrm{III}) = 143.8 .$$

the number of systems in each class being 6, 229, and 68, respectively.

It will be seen that in class II, where most of the spectroscopic binaries are found, the values of the C_i are quite uniform while the D_i increase with the period, and cannot be called constant. Hence, equation (9) is not satisfied but (10) is, and therefore, from (8), we can say that the average mass of a spectroscopic binary is independent of the period. Since (9) is not satisfied, (6) is not satisfied, and it follows that long-period spectroscopic binaries have not been formed from the short-period binaries through a secular decrease of mass.

The difference between C (III) and C (II) is probably due to the fact that K is small when $P>1000$ days, and the tendency will be to observe those systems for which K is larger than the average.

If we put $\sin^3 i=0.65$, $c=0.75$, and neglect e, it is found, from (7), that the average total mass of the spectroscopic binaries listed in Table VI is $3\times\odot$. We have already seen that the average mass of a

visual binary is $1.76 \times \odot$. In observing visual binaries the tendency will be to observe dwarfs that are close to the sun, while in the case of the spectroscopic binaries the tendency will be to observe distant giants. Since the giants are more massive than the dwarf stars, it follows that the difference between the masses of visual and spectroscopic binaries is small.

VI. CLOSE ENCOUNTERS

According to our first hypothesis in section i relating to the formation of binary stars, matter in a nebula condenses around two distinct nuclei and forms a visual binary with a long period. The period is certainly greater than several hundred years and probably runs into the thousands or tens of thousands. Binaries formed by the third hypothesis will also have very long periods.

TABLE VII

Spectroscopic Binaries		Visual Binaries	
P	e	P	e
$\leqq 3^{d}$	.05	20^{y}	.4
12	.2	75	.5
50	.3	135	.55
1000	.35	275	.6

The effect of a close encounter of a star with such a long-period system is either to break it up, to increase the period, or to decrease the period. If the original eccentricity were zero, the effect of an encounter necessarily would be to increase it. Now, it is well known that, statistically, the eccentricity increases with the period. Roughly speaking, the correlation runs about as shown in Table VII.

As may be seen from column 7 of Table VI, most of the spectroscopic binaries have extremely short periods, over 50 per cent being less than 12.59 days and over 70 per cent being less than 50.12 days. Several close encounters would have to take place in order to reduce the period of a binary of several hundred years to as low as only a few days, and, when the frequency of close encounters is taken into consideration, it is obvious that most short-period spectroscopic binaries could not have been formed in the manner just stated. Furthermore, it is doubtful if a binary could undergo many close encounters without being disrupted.

Let us now consider the possibility of systems with P greater than 55 days and up to about 25 or 100 years having been formed from longer-period binaries through close encounters.

The eccentricities and periods of 362 double stars are listed in Table VIII. The data for the spectroscopic binaries are taken from Beer's catalogue,[1] and that for the visual pairs is from W. van den Bos's table of orbits.[2] Of the 303 spectroscopic systems in Beer's list,

TABLE VIII

THE DISTRIBUTION OF BINARY STARS WITH RESPECT TO PERIOD AND ECCENTRICITY

e \ Group	1	2	3	4	5	6	7	8	9	10	11	Totals
log 10P	.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00	6.60	
P	$.4^d$	1.59	6.31	25.12	100.0	400	4.35^y	17.2	68.6	273	1160	
.1	1	10	67	26	7	9	3	.....	.....	.....	.....	123, *0*
.2	....	1	13	9	2	5	4	, *1*	, *7*	*2*	*1*	34, *11*
.3	....	1	3	15	3	1	2	4, *1*	, *3*	*3*	.....	29, *7*
.4	....	1	....	7	5	1	3	2, *2*	1, *10*	*6*	.....	20, *18*
.5	....	....	....	3	3	2	5	, *2*	, *7*	*14*	.....	13, *23*
.6	....	....	....	7	3	3	1	.....	, *7*	*13*	*3*	14, *23*
.7	....	....	....	3	.....	1	1	1, *1*	1, *1*	*4*	*1*	7, *7*
.8	....	....	....	.....	3	1	1	, *1*	, *3*	*7*	*2*	5, *13*
.9	....	....	....	.....	.....	1	.....	.....	, *3*	*4*	*1*	1, *8*
1.0	....	....	....	.....	.....	.....	.....	.....	, *1*	*3*	*2*	0, *6*
Totals..	1	13	83	70	26	24	20	7, *8*	2, *42*	*56*	*10*	246, *116*

e has not been determined for 49, and 8 others are also visual binaries, the value of e being determined by visual observations. These 8 are in van den Bos's table. One star in the list of 117 of van den Bos, Capella ($P = 104.02d$, $e = .0086$, from interferometer measures of P. W. Merrill), has been treated as a spectroscopic binary. The values of log 10P and P and of e denote the upper values in the group. The entries in the table in italicized numerals refer to visual systems, the others to spectrographic.

The binaries to the right of the heavy line in Table VIII, inserted between groups 5 and 6, all have periods greater than 100 days, and, except possibly for a few rare cases, should have originated as long-

[1] *Op. cit.*

[2] *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 149, 1926.

period systems. If the short periods of the binaries listed in groups 6, 7, and 8 are due to the effect of stellar encounters, then these systems must have had more encounters than those in groups 9, 10, and 11; and those in group 6 most of all.

Hence, since the average effect of encounters is to increase e, the average value of e as we go from the systems of group 11 to those of group 6 should increase. Actually, as we see from Table VII, the opposite is true. Referring to group 6 of Table VIII, we see that for more than one-half of the systems there $e \leqq .2$, while for more than one-third $e \leqq .1$. These latter stars are:

Boss No.	System	P	e
	H.D. 129132	101^d	.10
4934	v Sagittarii	137.9	.09
967	μ Persei	284	.06
1246	Capella	104.0	.02
	H.D. 184398	108.6	.05
5192	22 Vulpeculae	251.0	.04
1082	3 Camelopardi	121	.02
329	γ Phoenicis	193.8	.01
1773	A Carinae	195.3	.0

That the perturbations of a passing star could affect a binary in such a way as to make its orbit circular is possible but highly improbable. The fact that such a large portion of the binaries with periods between 100 and 400 days have very low eccentricities leads us to the conclusion that these systems, in the main, have not been formed from long-period systems through the effects of stellar encounters.

VII. EFFECTS OF A RESISTING MEDIUM

A resisting medium will reduce P, a, and, as we shall later see, e of a binary. MacMillan[1] has found that for a binary picking up matter in space, P varies inversely as the fifth power of M, the sum of the masses, and a varies inversely as the cube of M. Thus, if a star radiates no mass, binaries will undergo a secular decrease in P, a, and e.

Suppose, now, that a binary is picking up mass at a rate of dM_1/dt

[1] *American Mathematical Monthly*, **26**, 326, 1919.

and radiating it away at a rate of dM_2/dt simultaneously. For a star picking up matter,

$$P=\frac{c_1}{M^5}, \tag{11}$$

and, under the assumptions previously mentioned, for one losing mass by radiation,

$$P=\frac{c_2}{M^2}, \tag{12}$$

where c_1 and c_2 are constants. Let dP_1/dt and dP_2/dt be the rates of change of P due to the picking-up and radiation of mass, respectively. Differentiating (11) and (12) logarithmically gives

$$\frac{dP_1}{Pdt}=-\frac{5dM_1}{Mdt}, \tag{11'}$$

$$\frac{dP_2}{Pdt}=-\frac{2dM_2}{Mdt}. \tag{12'}$$

The total rate of change of the period, dP/dt, equals $dP_1/dt+dP_2/dt$. Hence, by (11') and (12'),

$$\frac{dP}{dt}=-\frac{P}{M}\left(5\frac{dM_1}{dt}+2\frac{dM_2}{dt}\right). \tag{13}$$

The term dM_1/dt is positive, while dM_2/dt is negative. If

$$-\frac{dM_2}{dt}<2.5\frac{dM_1}{dt}, \tag{14}$$

dP/dt is negative and P decreases.

Thus, under the assumptions with which we are working, *if a binary is radiating mass at a rate which is less than 2.5 times the rate at which it is picking it up, the period will decrease.*

It is found, similarly, that a will decrease if

$$-\frac{dM_2}{dt}<3\frac{dM_1}{dt}. \tag{15}$$

EFFECT ON THE ECCENTRICITY

We have already stated that a secular decrease of mass leaves the eccentricity invariant; we shall now discuss the change due to the

picking-up of matter. We shall assume that the particles of the resisting medium are mutually at rest, and that the center of gravity of the binary is moving through it with a velocity V_0. Also, let the two components of the binary be equal.

Let ξ, η, ζ represent a set of rectangular axes fixed in the medium. Let the ξ, η-plane be parallel to the plane of the orbit, and let the ξ-axis be parallel to the line joining the two bodies at the time of perihelion. The co-ordinates of the bodies are ξ_1, η_1, ζ_1; ξ_2, η_2, ζ_2, and their components of momentum are $m\xi_1'$, $m\eta_1'$, $m\zeta_1'$; $m\xi_2'$, $m\eta_2'$, $m\zeta_2'$. Since the matter picked up has no momentum, these components are not altered instantaneously, and the following equations must be satisfied:

$$\left.\begin{aligned} m(\xi_1')' &= -m'\xi_1' , \quad m(\eta_1')' = -m'\eta_1' , \quad m(\zeta_1')' = -m'\zeta_1' , \\ m(\xi_2')' &= -m'\xi_2' , \quad m(\eta_2')' = -m'\eta_2' , \quad m(\zeta_2') = -m'\zeta_2' . \end{aligned}\right\} \tag{16}$$

Since the matter in the medium is at rest with respect to the ξ, η, ζ-axes, the rate at which each of the bodies is sweeping up matter is proportional to its absolute velocity. Hence, we may write

$$\left.\begin{aligned} m' &= mhV_i , \\ V_i^2 &= \xi_i'^2 + \eta_i'^2 + \zeta_i'^2 , \qquad (i = 1 \text{ or } 2) . \end{aligned}\right\} \tag{17}$$

The term mh is essentially constant over an interval of time such as one period; its value is equal to the amount of matter which each body picks up in one period divided by the distance which it traverses in the same interval of time. Substitution of the first of (17) in (16) shows that the resistance is proportional to the square of the absolute velocity and that its direction is opposite to that of the absolute motion.

The equations of motion in the absolute co-ordinates are

$$\left.\begin{aligned} \xi_1'' &= -\frac{k^2 m(\xi_1 - \xi_2)}{r^3} - h\xi_1' V_1 , \\ \xi_2'' &= +\frac{k^2 m(\xi_1 - \xi_2)}{r^3} - h\xi_2' V_2 , \end{aligned}\right\} \tag{18}$$

and similar equations in the other co-ordinates. Let

$$x=\xi_1-\xi_2\,, \qquad y=\eta_1-\eta_2\,, \qquad z=\zeta_1-\zeta_2\,. \tag{19}$$

By means of (19) we find, from (18), that

$$\left.\begin{aligned} x''&=-\frac{2k^2mx}{r^3}-X\,,\\ y''&=-\frac{2k^2my}{r^3}-Y\,,\\ z''&=-\frac{2k^2mz}{r^3}-Z\,, \end{aligned}\right\} \tag{20}$$

where $X=h(\xi_1'V_1-\xi_2'V_2)$ and $Y=h(\eta_1'V_1-\eta_2'V_2)$, etc. Equations (20) are the equations of motion in the relative co-ordinates. The quantities X, Y, Z are the components of the disturbing acceleration, resolved along the co-ordinate axes. According to F. Tisserand,[1] the variation in e due to these components is

$$\left.\begin{aligned} \frac{de}{dt}=\frac{\epsilon}{na}\,[&(\cos E+\cos v)(X\sin v-Y\cos v)\\ &-\sin v(X\cos v+Y\sin v)]\,, \qquad \epsilon=(1-e^2)^{\frac{1}{2}}\,, \end{aligned}\right\} \tag{21}$$

where n is the mean motion, E is the eccentric anomaly, and v is the true anomaly. It will be noticed that Z does not enter in this equation.

Let α, β, γ be the components of V_0 referred to the ξ, η, ζ-axes. Then

$$2\alpha=\xi_1'+\xi_2'\,, \qquad 2\beta=\eta_1'+\eta_2'\,, \qquad 2\gamma=\zeta_1'+\zeta_2'\,. \tag{22}$$

It is found from (22) and (19), after the latter have been differentiated with respect to t, that

$$\left.\begin{aligned} 2\xi_1'&=2\alpha+x'\,, & 2\xi_2'&=2\alpha-x'\,,\\ 2\eta_1'&=2\beta+y'\,, & 2\eta_2'&=2\beta-y\,,\\ \zeta_1'&=\zeta_2'=\gamma\,, & &(\text{since } z'\equiv 0)\,. \end{aligned}\right\} \tag{23}$$

[1] *Traité de mécanique céleste*, **4**, 229, 1896.

From the properties of Keplerien motion we know that

$$\left.\begin{aligned} x' &= -\frac{na\sin E}{1-e\cos E}, \\ y' &= \frac{\epsilon\, na\cos E}{1-e\cos E}, \\ \sin v &= \frac{\epsilon\sin E}{1-e\cos E}, \\ \cos v &= \frac{\cos E-e}{1-e\cos E}, \\ nt &= E-e\sin E. \end{aligned}\right\} \tag{24}$$

By means of (23) and (24) it is possible to eliminate t, v, and the derivatives of the absolute co-ordinates from (21). It is found, after performing the various substitutions, that δe, the change in e in one period, is

$$\delta e = -\frac{\epsilon^2 h(G+H)}{n},$$

where

$$\left.\begin{aligned} G &= \int_0^{2\pi}(V_1+V_2)\cos E\, dE, \\ H &= \frac{1}{na\epsilon^2}\int_0^{2\pi}(V_1-V_2)\,[\beta\epsilon(1+\cos^2 E-2e\cos E)-\alpha\sin E\cos E]dE. \end{aligned}\right\} \tag{25}$$

We shall now show that G is always greater than zero, proving first, however, that when regarded as functions of E

$$V_1(E)+V_2(E)\geqq V_1(E+\pi)+V_2(E+\pi), \qquad \left(+\frac{\pi}{2}\geqq E\geqq -\frac{\pi}{2}\right). \tag{26}$$

Let $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ be vectors representing the velocity of the two components with respect to the center of gravity; $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ vectors representing their absolute velocities; and $\boldsymbol{V}_0$ a vector representing the absolute velocity of the center of gravity. Then

$$\boldsymbol{V}_1=\boldsymbol{U}_1+\boldsymbol{V}_0, \qquad \boldsymbol{V}_2=\boldsymbol{U}_2+\boldsymbol{V}_0.$$

Since the components are equal in mass, $\boldsymbol{U}_1 = -\boldsymbol{U}_2$ by symmetry. From the solution of the problem of two bodies we know that

$$|\boldsymbol{U}_1(E)| \geqq |\boldsymbol{U}_1(E+\pi)| , \qquad \left(+\frac{\pi}{2} \geqq E \geqq -\frac{\pi}{2}\right) .$$

By the symmetry of the ellipse the two vectors in the foregoing equation are parallel. Thus, the vectors $\boldsymbol{U}_1(E)$, $\boldsymbol{U}_2(E)$, $\boldsymbol{U}_1(E+\pi)$, and $\boldsymbol{U}_2(E+\pi)$ are all parallel to each other. Let c and d be the magnitude of the components, respectively, of $\boldsymbol{V}_0$ when resolved along them and perpendicularly to them. Let u be the magnitude of $\boldsymbol{U}_1(E)$ and w be the magnitude of $\boldsymbol{U}_1(E+\pi)$. Inequality (26) now becomes

$$[(c+u)^2+d^2]^{\frac{1}{2}}+[(c-u)^2+d^2]^{\frac{1}{2}} \geqq [(c+w)^2+d^2]^{\frac{1}{2}}+[(c-w)^2+d^2]^{\frac{1}{2}} , \qquad (u^2 \geqq w^2) .$$

By virtue of the relation between u^2 and w^2 this can be reduced to another inequality,

$$(c^2+d^2)^2(u^2-w^2)^2 \geqq (c^2-d^2)^2(u^2-w^2)^2 ,$$

which is always satisfied. Hence, equation (26) is also satisfied. Because of this and because of the known properties of the function cos E it is easy to show that the integral G is always greater than zero. If H were small compared to G then the eccentricity would decrease continually, even if H were negative. From the second of (25) it follows that H equals zero if both α and β are zero, i.e., if the motion of the center of gravity is perpendicular to the orbital plane.

From the second of (17), (23), and the first two of (24) it is found that

$$\left.\begin{aligned} V_1 &= \frac{(I^2+J)^{\frac{1}{2}}}{2(1-e\cos E)^{\frac{1}{2}}} , \\ V_2 &= \frac{(I^2-J)^{\frac{1}{2}}}{2(1-e\cos E)^{\frac{1}{2}}} , \end{aligned}\right\} \qquad (27)$$

where $I^2 = n^2a^2+4V_0^2+(n^2a^2-4V_0^2)e\cos E$, $J = 4na(\epsilon\beta\cos E-\alpha\sin E)$, $V_0^2 = \alpha^2+\beta^2+\gamma^2$. Expansion of V_1 and V_2 in powers of J/I^2 shows that the ratio of (V_1+V_2) to (V_1-V_2) is of the order of na/V_0 (α, β, γ, and V_0 are all considered to be of the same order).

Hence, from (25), the ratio of G to H is of the order of $(na/V_0)^2$. The term na is approximately the relative velocity of the two components of the binary, and is indeed equal to it, as may be seen from the first two of (24), if e is zero. It varies inversely as the cube root of the period P. For a binary whose total mass is 2.5 times that of the sun, $na = 290$ km/sec. if $P = 1$ day; 62 if $P = 100$ days; 30 if $P = 1000$ days; and 9 if $P = 100$ years. The term V_0 will, on the average, probably be less than 15 km/sec.

Thus, if it is true that the periods of binaries are being reduced through the gathering of interstellar matter, then the eccentricities of short-period spectroscopic binaries should be small. In the case of visual binaries whose periods are of the order of 100 years or more we may expect a wide divergence in the values of the eccentricities since the absolute velocity of the center of gravity is comparable to the relative velocity of the components; the effects of close encounters with other stars probably has more influence on the eccentricities of these systems than does the picking-up of matter.

SOME NUMERICAL RESULTS

It may be of interest to compute the time required for a hypothetical visual binary to evolve into a spectroscopic binary. We shall assume that (*a*) each component has the mass of the sun; (*b*) the mass radiated by each star is always equal to the mass it picks up, and this is equivalent to the amount of energy radiated by the sun at the present time (assuming that 1 gm $= 9 \times 10^{20}$ ergs).

Since two suns are losing and gaining mass,

$$\frac{dM_1}{dt} = -\frac{dM_2}{dt} = 2.6 \times 10^{20} \text{gm/yr.} \tag{28}$$

Substituting (28) in (13), putting $M = 4 \times 10^{33}$ gm, and integrating, gives

$$P = P_0 e^{-2 \times 10^{-13} t},$$

where P_0 is the value of the period when $t = 0$, t being expressed in years.

Putting $t = 7 \times 10^{13}$, we find that $P_0/P = 1.2 \times 10^6$. Thus, the period, if it were originally 10,000 years, would be reduced to about 3 days in 70,000,000,000,000 years.

From (12′) it is found that the change of the period of a spectroscopic binary due to a secular decrease of mass is of the order of a few millionths of a second per year or less, and from (11′) it is seen that the change due to picking up matter would be also undetectable unless this process took place at a rate which is enormous compared to the loss of mass by radiation.

To determine the change in the eccentricity of the binary discussed here we shall make the further assumption that it is at rest relative to the medium. Then $\alpha=\beta=\gamma=V_0=0$, and, because of (25) and (27),

$$\delta e=-ah(1-e^2)\int_0^{2\pi}\cos E\left(\frac{1+e\cos E}{1-e\cos E}\right)^{\frac{1}{2}}dE\,.$$

The integrand of the foregoing expression can be expanded in powers of e. The terms containing even powers of e will carry odd powers of $\cos E$ as factors, and, as these vanish in the integration, it follows that the integral is expansible in odd powers of e. Expanding and integrating, we obtain

$$\delta e=-ah\pi(1-e^2)(e+\tfrac{3}{8}e^3+\tfrac{1}{6}\tfrac{5}{4}e^5+\cdots)\,. \tag{29}$$

If dm_1/dt is the average rate at which each of the components is picking up matter and ds is the element of arc length, then, since $V=ds/dt$, it is found from the first of (17) that

$$\frac{dm_1}{dt}=\int_0^P\frac{dm}{P}=\frac{mh}{P}\int_0^P ds\,.$$

The last integral on the right-hand side is the expression for the ellipse through which each component moves. If the length of the primeter is expanded as a power series in even powers of e, then

$$\frac{Pdm_1}{dt}=mh\pi a(1-\tfrac{1}{4}e^2-\tfrac{3}{6}\tfrac{}{4}e^4-\cdots)\,. \tag{30}$$

Eliminating h from (29) by means of (30), it is found, since the average value of de/dt equals $\delta e/P$, that

$$\frac{1}{e}\frac{de}{dt}\left[\frac{(1-\tfrac{1}{4}e^2-\tfrac{3}{6}\tfrac{}{4}e^4-\cdots)}{(1-e^2)(1+\tfrac{3}{8}e^2+\tfrac{1}{6}\tfrac{5}{4}e^4+\cdots)}\right]=-\frac{1}{m}\frac{dm_1}{dt}\,.$$

Expanding the bracket in even powers of e and integrating gives, neglecting terms of the sixth order and higher,

$$\log \frac{e}{e_0} + \tfrac{3}{16}(e^2 - e_0^2) - \tfrac{9}{256}(e^4 - e_0^4) = -\frac{t}{m}\frac{dm_1}{dt},$$

e_0 being the value of e when $t = 0$. If t is expressed in years, then its coefficient, for the system considered here, is equal to -0.65×10^{-13}. Suppose that $e_0 = 0.5$. It is found that e/e_0 will equal 0.1, or that e will equal .05 after an interval in t of about 1.5×10^{13} years.

Consider, then, the evolution of a long-period binary which satisfies (14). P, a, and e will decrease continually, and the mass ratio will tend toward unity, since the less massive star swings in a larger orbit and picks up more matter. If in the last stages of its existence it was in a rich region of space, the binary would end up as a spectroscopic system of large mass, high temperature, low density, and early spectral type, such as the O8 system H.D. 1337 (Boss 46). If, on the contrary, it was in a relatively rare region of space, so that its mass remained nearly constant, it would end up with a small mass, low temperature, high density, and late spectral type, such as the F8 system W Ursae Majoris.

The two components of the binary may originally have had rotational motions, and while tidal forces would tend to dampen them, they might, in certain cases, persist. Certain irregularities in the light-curves of some eclipsing binaries may be due to the fact that the components are rotating in planes which do not coincide with the orbital plane.

The data in section iv indicates that tidal forces will disrupt one of the components when the distance between the surfaces is about as large as their diameters ($\sigma = 0.5$). Just what will happen to the disrupted star is not easy to say. MacMillan believes that it might form a gaseous ring about the other.[1] The material in this ring may be gradually absorbed by the remaining star, resulting in an early type star, such as O or B, with a high rotational velocity. Another possibility is that the removal of the surface layers of a star may cause a nova.

[1] See, in this connection, an article by O. Struve, *op. cit.*, **73**, 94, 1931.

VIII. INTERSTELLAR MATTER

Whether or not equation (14) is satisfied in general by binaries, our present knowledge of interstellar conditions does not permit us to say definitely.

On the assumption that a star loses 1 gm of mass for each 9×10^{20} ergs emitted, we find that the mass of the sun is decreasing at the rate of 4.2×10^{12} gm/sec. MacMillan[1] found that the rate of increase of mass of a single star in passing through a nebulous region is given by

$$\frac{dm}{dt}=\frac{2\pi k^2 mR\rho}{V}, \tag{31}$$

where k^2 = the gravitational constant, m = the star's mass, R = the star's radius, ρ = the density of the medium, and V = the relative velocity between the star and the nebula. For the sun, $R=7\times10^{10}$, $m=2\times10^{33}$, $V=2\times10^{6}$ (20 km/sec.), all units being in the C.G.S. system. Substituting these values in (31), and putting $dm/dt=4.2\times10^{12}$, we find that ρ must equal 1.4×10^{-19} if the sun is to gather in as much mass as it is radiating away.

The eclipsing binary H.D. 1337 (Boss 46) is the hottest and most massive system whose absolute dimensions are known. From J. A. Pearce's[2] data we have

$$\begin{aligned} R_1 &= 23.8\times\odot, \\ R_2 &= 15.5\times\odot, \\ m_1+m_2 &= 70\times\odot, \\ \textit{Energy emitted} &= 10^4\times\odot. \end{aligned}$$

The two components being practically in contact, we shall put $R=40\times\odot$. It is found that in order that the total mass should not vary this system must pass through a nebula of density of

$$\rho=1.7\times10^{-23}V.$$

On the assumption that $V=10$ km/sec.,

$$\rho=1.7\times10^{-17}.$$

[1] *American Mathematical Monthly*, **26**, 326, 1919.

[2] *Publications of the Dominion Astrophysical Observatory*, **3**, 275, 1927.

When the high luminosity of such stars as H.D. 1337 is taken into account, it develops that they must be exceedingly rare in space. In general, then, a single star must pass through diffuse matter whose density is of the order of 10^{-19} or 10^{-18} in order that its mass should remain constant.

We have assumed that for the sun $R=7\times10^{10}$ cm, the radius of the photospheric shell. It may well be that a particle of interstellar matter which is intercepted by the corona will fall into the sun. In that case the density required to sustain the sun would be of the order of 10^{-20} gm/cm^3.

In discussing the densities required to sustain binaries another factor enters. Equation (31) was derived for single stars on the assumption that every particle of matter in the nebula which is attracted by the star, and whose perihelion distance is less than the radius of the star, would fall into it. The semi-major axis of a binary is larger than the sum of the radii of the components, and because of their rapid velocity about each other more matter must be picked up than if the components were pursuing independent paths. Thus, densities of the order of 10^{-20}, or 10^{-21}, or even lower may be sufficient to maintain the masses of binaries.

This does not mean that the density of interstellar matter must be uniformly of about these orders. The existence of luminous and non-luminous nebulae suggests that the density varies widely from point to point. If conditions were right, stars could pick up enough matter in their slow journeys through dense nebulosity to last them through the faster flights through the rarer parts of space. Thus, the life of a star may be of indefinite duration. From the results of the preceding section it follows that if a binary should gain and lose mass alternately, P, a, and e would be reduced.

Interstellar matter may be considered to be of three kinds: (1) atomic, *Ca*, *Na*, etc.; (2) fine dust; (3) large bodies, meteors, etc.

J. C. Kapteyn[1] found that the average "effective" mass of a star within the galaxy, i.e., the total mass of a given volume of galactic space divided by the number of luminous stars therein, to be $1.6\times\odot$. We have already seen that the average mass of one component of a visual binary to be $.88\times\odot$ (the number of spectroscopic binaries

[1] *Astrophysical Journal*, **55**, 302, 1922.

which occur in a given volume of galactic space is very small in comparison with the number of visual binaries). It will be shown in Table IX that most of the stars within 10 parsecs of the sun are dwarfs later in spectral type than the sun, and, therefore, presumably less massive than it. Hence, from this interpretation of Kapteyn's results it appears that the amount of non-luminous matter in the galaxy is of about the same order as that contained within the luminous stars. Eddington[1] obtains a similar result, and deduces 10^{-23} gm/cm^3 to be the average density of interstellar matter.

That interstellar matter of atomic size does exist is well known because of the recent work on the stationary H and K line of *Ca* and the D lines of *Na*. Eddington[2] and Struve[3] find densities of the order of 10^{-24} for the *Ca* and *Na*. Other free atoms may exist, of course, whose presence it would be difficult or impossible to detect. J. S. Plaskett and J. A. Pearce[4] find that the *Ca* and *Na* atoms partake of the rotation of the galaxy and, since the rotational terms for the stationary lines are one-half that of the stars, deduce that these free atoms cannot form a localized cloud around the star. Nevertheless, this matter may be concentrated into many small clouds which are themselves uniformly distributed.

Further evidence of interstellar matter in the galactic system is obtained from observations of absorption of light. R. J. Trumpler[5] interprets the absorption as being due to dust particles whose masses are of the order of 2×10^{-19} gm.

With respect to the larger particles, meteors, etc., practically nothing is known since it would take an exceedingly large amount to cause any noticeable absorption.

The large amounts of nebulosity in the galactic system, especially the dark, shows that there exist numerous portions of space where the density of interstellar matter is high enough to produce the massive O- and B-type stars. The close relation between these stars and adjacent or surrounding nebulosity is well known.

[1] *Monthly Notices of the Royal Astronomical Society*, **75**, 375, 1915.

[2] Proceedings of the Royal Society, A, **111**, 424, 1926.

[3] *Monthly Notices of the Royal Astronomical Society*, **89**, 567, 1929.

[4] *Ibid.*, **90**, 243, 1930.

[5] *Astronomical Society of the Pacific*, **42**, 214, 1930.

Summarizing these results, then, we might state:

i) We cannot as yet determine from observation alone whether or not, in general, single stars and binaries are picking up enough interstellar matter to compensate for the loss by radiation, if any.

ii) For the galaxy, the amount of interstellar matter may be comparable in mass to the amount contained within the luminous stars.

iii) There probably exist nebulae of density high enough to form stars of type B and O.

According to the cosmological ideas of MacMillan, the energy of a star is obtained by the annihilation of its mass, and radiant energy in turn is condensed, so to speak, into matter. If the foregoing processes are taking place in our galaxy, and the mass of the galaxy were constant, it would be reasonable to suppose that it would tend to a state of equilibrium wherein the amount of matter contained in the stars and the amount in the form of nebulae and other interstellar matter would be constant; i.e., the amount of matter lost by radiation by the stars would equal the amount swept up.

Suppose, however, that the mass of the galaxy is not constant; that it is radiating more energy into space than it is receiving from other galactic systems. In that case the amount of matter in the stars and in the form of interstellar matter would gradually decrease, but again we may have a kind of equilibrium whereby the amount of matter picked up by the stars would be only slightly less than the amount radiated.

Let us assume that the rate of loss of mass per gram of the galaxy is the same as that of the sun. The galaxy is probably losing mass at a rate much smaller than this, since interstellar matter is not radiating at all, and because the actual number of massive stars of early spectral type O, B, and A forms only a small percentage of all the stars in the galaxy. Thus, of the 77 stars listed in F. Schlesinger's *General Catalogue of Stellar Parallaxes* (1924) that are within 10 parsecs of the sun, and whose spectral types are known, the distribution is as given in Table IX. With the exception of one star, all of the foregoing 77 lie on the main sequence.

It is known that the rate of emission of energy of a star increases

with the mass. From Eddington's mass-luminosity diagram[1] we find that this relation can be expressed as follows:

$$\frac{dm}{dt} = -\lambda m^{\nu}, \tag{32}$$

If m does not change very much, λ and ν can be considered constant. It is found that $\nu = 4.8$ for $m = 0.2 \times \odot$, 3.6 for $m = \odot$, and 1.6 for $m = 10 \times \odot$. Integrating (32) gives

$$t = \frac{\nu - 1}{\lambda m_0^{\nu-1}} (\mu^{\nu-1} - 1), \qquad (\nu \neq 1), \tag{33}$$

where $\mu = m_0/m$, and m_0 is the present mass of the sun ($t = 0$).

TABLE IX

	Oe–B8 B	B9–A3 A	A5–F4 F	F5–G2 G	G5–K4 K	K5–Md M	Total
No.	0	4	5	13	31	24	77
Per cent	0	5.2	6.5	16.9	40.2	31.2	100

For the sun $dm/dt = -1.3 \times 10^{20}$ gm/yr., and $m_0 = 2 \times 10^{33}$ gm. Substituting these values in (32) and (33), (33) becomes

$$t = (\nu - 1) 1.5 \times 10^{13} (\mu^{\nu-1} - 1). \tag{34}$$

We have found in section vii, under the assumption that a binary picks up matter at the same rate at which it is radiating it away, that 7×10^{13} years is sufficient to change it from a visual to a spectroscopic binary. Putting t in (34) equal to this number and $\nu = 3.6$, it is found that the sun, in this interval of time, will lose .32 of its mass. For $\nu = 3$ and 4 the corresponding decreases are .45 and .27. In view of the fact that the entire galaxy is not losing mass at the same rate as the sun, it follows that the mass of the galaxy will not change very much in the time required to transform a long-period binary into a short-period binary.

[1] *The Interior Constitution of the Stars*, Fig. 2, p. 153, 1926.

SUMMARY

The possibility that short-period spectroscopic binaries have evolved into long-period systems has been investigated statistically. It was found that neither contraction, secular decrease of mass, nor close encounters are sufficient to cause the necessary increase in P, a, and e. The possibility that binaries whose periods are less than 25 years have been formed from long-period binaries by the effects of stellar encounters was shown to be small, in view of the large number of systems in the former class with small eccentricities.

The effect of a resisting medium was discussed, and it was shown that P, a, and e would decrease even if the binary is losing as much mass by radiation as it is picking up. While the average interstellar density of the galaxy is perhaps not high enough to maintain equilibrium of mass, there probably exist many regions which are dense enough to form massive stars.

The uniform average mass of binaries, independent of period, as deduced from column 8 of Table VI, and the uniform increase of e with P, as shown in Tables VII and VIII, lead to the conclusion that binary stars, as a rule, have had a common origin; and any apparent separation of binaries into two classes for which P is less than 55 days and P is greater than 55 days is probably due to the observational methods employed.

CONCLUSION

Irrespective of the truth of any theories on the relation between mass and energy and of the density of interstellar matter, one of the following three conditions must apply to any given binary:

A. The system is not losing any mass because of the radiation of energy.

B. It is radiating mass less than 2.5 times as fast as it is gathering it in.

C. It is radiating mass more than 2.5 times as fast as it is gathering it in.

If either condition A or B is satisfied, then P, a, and e are decreasing. If C is satisfied, then, assuming that there are no forces operat-

ing except those already considered, there should be a statistical correlation between the period P and the total mass M of the form

$$M = \frac{D_1}{P^{\frac{1}{3}}},$$

where D_1 is a constant. But we have seen in section v that such is not the case. Therefore, condition A or B must be satisfied. Thus, if there are no forces acting on a binary which tend to increase P, a, and e other than those due to tides, secular decrease of mass, and close encounters, then, in general, P, a, and e of a binary are decreasing.

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ON THE CONTOURS OF NARROW ABSORPTION LINES

REMARKS CONCERNING RECENT PAPERS BY R. v. d. R. WOOLLEY

By ALBRECHT UNSÖLD

ABSTRACT

I. A simple approximate method for calculating the widening of spectral lines by a grating spectrograph is developed. It is shown that the deviations from theory found recently by R. v. d. R. Woolley in observations of atmospheric oxygen lines are due to this instrumental effect.

II. Woolley's measures of contours of solar lines are affected by the same systematic error.

III. 1. It is shown that in the range of line-widths, where Doppler effect comes into play, the actual line-contours cannot be fully resolved with a grating of the usual size.

2. Following Minnaert and Mulders, the limitations of the Russell-Adams calibration are discussed.

3. The exchange of radiation in solar lines is considered. It is shown that in the case of penultimate lines interlocking does not make quantitative spectral analysis impossible. Interlocking may give a mechanism for producing central intensity, which avoids the difficulties of the collision theory.

INTRODUCTION

Recently Dr. R. v. d. R. Woolley has published in the *Astrophysical Journal* several papers[1] which deal with the contours of rather narrow Fraunhofer lines. He obtains results which are, to a considerable extent at least, in marked contradiction to current theory. In the present note I hope to show that these results of Woolley are simply due to the neglect of the influence of finite resolving power of the spectrograph. Then we shall consider more closely some of the theoretical opinions brought forward by Woolley and shall show that the interpretation of line-contours is not at all as uncertain as some authors seem to think.

I. THE DISTORTION OF LINE-CONTOURS BY A GRATING SPECTROGRAPH; RELATION BETWEEN LINE-WIDTH AND "NUMBER OF ATOMS"

Let us first consider Woolley's second paper, which investigates the width of atmospheric oxygen lines as a function of effective air mass (derived from zenith distance). Since these lines are broadened

[1] *Op. cit.*, **72**, 256, 1930; *ibid.*, **73**, 185 and 194, 1931; subsequently denoted by No. 1, No. 2, and No. 3.

chiefly by Lorentz collision-damping, one should expect that—as in the case of radiative damping—their width increases proportionally to the square root of the effective air mass. Instead of this, Woolley gets an increase about $\sim$ (air mass)$^{3/8}$.

The spectra from which this result was derived were obtained in the first order of the Mount Wilson 75-foot spectrograph, which has a resolving power of $\sim 77,500$. At λ 6910, the mean wave-length of these oxygen bands, this corresponds to a difference in wave-length of $\Delta\lambda_0 = 6910/77{,}500$ A $= 89$ milliangström. Many of Woolley's lines at $r = 3/4$[1] have a width $(\lambda - \lambda_0)$ of the same order of magnitude and consequently, as every spectroscopist knows, Woolley's supposition that "these lines are so wide that the finite resolving power of the instrument does not affect measurements of their width" cannot be true with respect to his narrower lines.

Let us now calculate the change in line-width produced by a grating spectrograph and measured at a point where the contrast $K = 1 - r$. Rigorous formulae for treating this problem have been given by J. A. Carroll,[2] but for our purpose we shall prefer a somewhat simplified treatment which avoids the troublesome numerical integrations of the strict procedure.

The contour of a strictly monochromatic line (wave-length λ), produced by a grating of resolving power R = order of spectrum $\times$ number of lines, is, to a very high degree of approximation, given by

$$I = \text{Const.} \left(\frac{\sin \pi \dfrac{\Delta\lambda}{\Delta\lambda_0}}{\pi \dfrac{\Delta\lambda}{\Delta\lambda_0}} \right)^2, \qquad (1)$$

where $\Delta\lambda_0 = \lambda/R$ and $\Delta\lambda$ is the distance from the center of the line. In order to be able to get true line-widths from the observed ones without the use of numerical integration, we shall approximate this expression by fitting to it a Gauss curve (cf. Fig. 1), namely,

$$I = \frac{1}{\beta\sqrt{\pi}} e^{-\left(\frac{\Delta\lambda}{\beta}\right)^2}, \qquad (2)$$

[1] I.e., measured where the intensity is three-fourths of the neighboring continuous spectrum.

[2] *Monthly Notices of the Royal Astronomical Society*, **88**, 154, 1927.

with $\beta = 0.60\ \Delta\lambda_0$. (The integral over I has been normalized to 1.) Let the contour of an absorption line, as observed with an instrument of infinite resolving power, be given by[1]

$$I' = I_0' \left\{ 1 - a e^{-\left(\frac{\Delta\lambda}{a}\right)^2} \right\}. \tag{3}$$

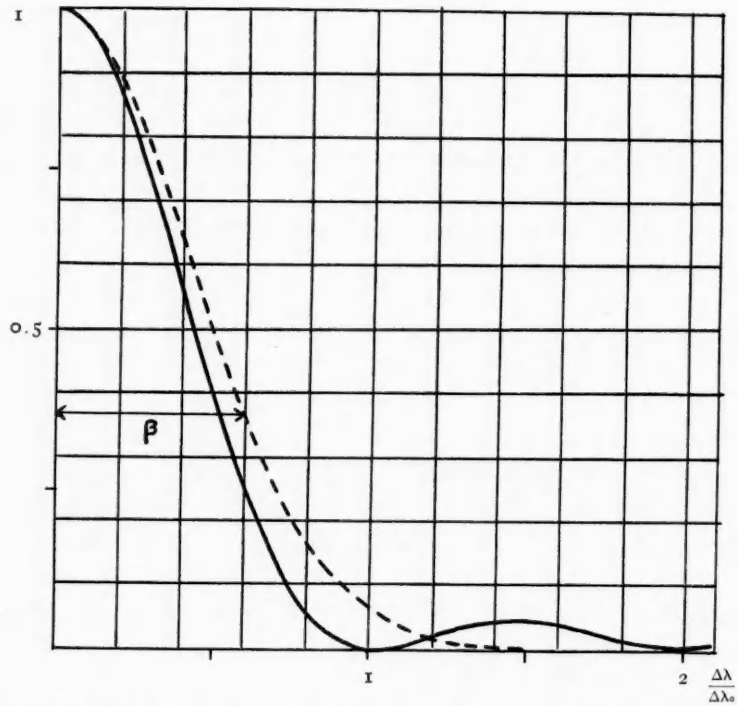

FIG. 1.—True contour of spectral line produced by grating (continuous line) and approximation to it (dotted line).

Then the contrast K in the line as observed with our real spectrograph is given by

$$K(\Delta\lambda) = a \int_{-\infty}^{+\infty} e^{-\left(\frac{\Delta\lambda + x}{a}\right)^2} \cdot \frac{1}{\beta\sqrt{\pi}} e^{-\left(\frac{x}{\beta}\right)^2} dx = \frac{a}{\sqrt{1 + \left(\frac{\beta}{a}\right)^2}} e^{-\frac{\Delta\lambda^2}{a^2+\beta^2}}. \tag{4}$$

[1] Using this simplified expression, we follow Carroll's paper. I think that as long as we use this expression for the line-contour it is of no advantage to use equation (1) instead of the much more convenient equation (2).

As a test of the accuracy of our simplified formula (2) let us compare the central depth of a line with originally black center ($a=1$) computed (1) from equation (4) as $[1+(\beta/\alpha)^2]^{-\frac{1}{2}}$ and (2) as given by Carroll from the numerical integration of the strict formula as a function of $\alpha/\Delta\lambda_0$. Especially for narrow lines, where the influence of the secondary diffraction maxima is reduced, equation (4) proves to be a reasonable first approximation (see also the preceding footnote).

TABLE I

$\frac{\alpha}{\Delta\lambda_0}$	$\left[1+\left(\frac{\beta}{\alpha}\right)^2\right]^{-\frac{1}{2}}$	Carroll's Accurate Expression
5.....	0.99	0.94
2.....	.96	.86
1.....	.86	.81
0.5.....	.64	.63
0.2.....	0.32	0.33

We give in Figure 2 as a first application the true (full-line) and observed (dashed-line) contours of a line with $\alpha=53.5$ milliangström and $a=1$ (black center) observed at λ 6910 in the first order of a grating of resolving power 77, 500, so that

$$\beta=0.6\,\frac{6,910}{77,500}\cdot 10^3=53.5 \text{ milliangström}. \qquad (5)$$

This line has at $r=9/10$ an observed width of 107 milliangström and is only slightly weaker than the faintest lines of Woolley's paper No. 2. As Figure 2 shows, the influence of the grating is very great.

Let us now consider two lines with $\alpha_1=200$ and $\alpha_2=80$,[1] corresponding to the strongest and weakest lines of Woolley's paper No. 2 (right and left end of his curve, Fig. 1). We assume that the lines were originally black, i.e., $a=1$. The true and observed widths at $r=9/10$ and $r=3/4$ calculated from equations (3) and (4) are then as shown in Table II, if we put, according to (5), $\beta=53.5$. In order to correct for the influence of the grating we must increase the ratio of the $(\Delta\lambda)^2$ of the stronger to the weaker line by a factor of

$$\left\{\frac{304}{312}:\frac{121}{140}\right\}^2=1.27 \text{ for } r=\frac{9}{10} \text{ and } \left\{\frac{235}{241}:\frac{94}{106}\right\}^2=1.20 \text{ for } r=\frac{3}{4}.$$

[1] All the α, $\Delta\lambda$, etc., are expressed in milliangström.

The average is 1.23.[1] Thus, the left end of Woolley's curve in his Figure 1 (paper No. 2) should be lowered by log 1.23 = 0.09, corresponding to 5.5 mm. This brings the experimental and theoretical curves much nearer to parallelism than they were before. In addition to the effect of the grating we have next to take into account the effect of slit-width. This is not stated in Woolley's paper; if we assume it to be ~5/100 mm, this would correspond on the plate to

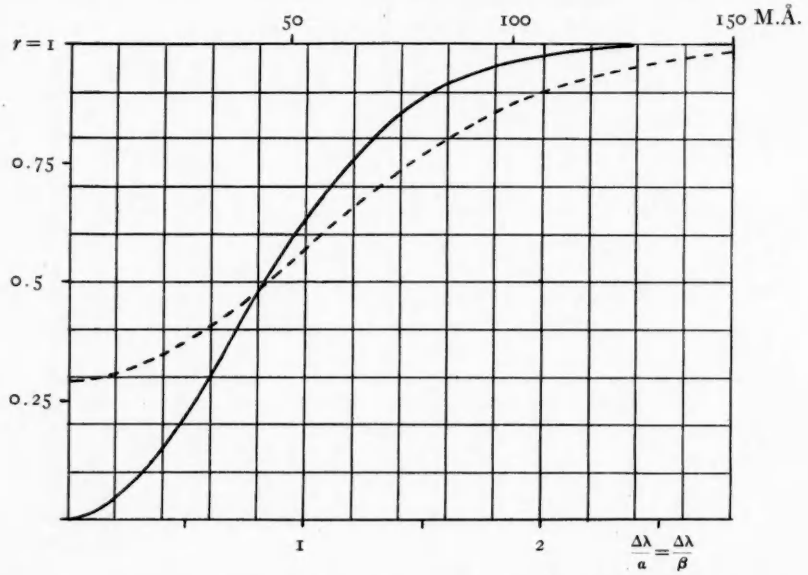

FIG. 2.—True contour of spectral line (continuous) and observed contour (dotted).

~35 milliangström, which would give an increase in the "effective" β of ~20 per cent.[2] Next we should consider the widening of the lines by small irregularities of the grating, but this effect cannot be calculated. One may guess that it is not larger than that given by the slit. Summing up these effects, we cannot doubt that all the deviations from the $\sqrt{N}$-law found in Woolley's paper No. 2 are of purely instrumental origin.

[1] In forming this average we follow Woolley's paper. As we shall see later, the physical foundations of such averaging processes are very weak. On the other hand, a detailed reduction is not yet possible from lack of data (cf. below).

[2] For more accurate calculations see P. H. van Cittert, *Zeitschrift für Physik*, **65**, 547, 1930 and **69**, 298, 1931, where former literature is also given.

An empirical calculation of the effective β of the Mount Wilson 75-foot spectrograph would be possible from observations of narrow emission lines or—under the assumption of $a=1$—even from the observed central intensities of Woolley's lines, but these are not stated in his paper. Another direct evidence that Woolley's lines are affected instrumentally is obtained by plotting his (reduced) widths (Table II) for $r=9/10$ against those for $r=3/4$. If the lines were similar, i.e., could be transformed into one another merely by changing the scale in λ (as must be assumed, if Woolley's way of reducing shall have any sense), the reduced values of the $(\Delta\lambda)_{r=\frac{9}{10}}$ should equal the corresponding $(\Delta\lambda)_{r=\frac{3}{4}}$. But in reality his results give about $(\Delta\lambda)_{r=\frac{9}{10}}=0.78(\Delta\lambda)_{r=\frac{3}{4}}$. As can be seen from our previous

TABLE II

		$a_1=200$	$a_2=80$
$r=9/10$	True width	304	121
	Observed width	312	140
$r=3/4$	True width	235	94
	Observed width	241	106

considerations, an effect of exactly this sort is produced by the grating: for some definite finite value of $(\Delta\lambda)_{\frac{9}{10}}$ the $(\Delta\lambda)_{\frac{3}{4}}$ becomes zero while for wide lines both values are equal; the inclination of the curve connecting the two extreme cases is exactly of the character shown by Woolley's observations.

In concluding this section I should like to point out that almost two years before Woolley's paper appeared I showed[1] by comparing the spectra of the first and third orders that the 75-foot spectrograph used by Woolley does not fully resolve in the first order the Ba^+ lines λ 6496.9 and λ 5853.7 of Rowland intensities 4 and 5 (the latter estimate is probably too high).

II. WOOLLEY'S MEASUREMENTS OF FAINT SOLAR LINES

In his first paper Woolley deals with the contours of Fraunhofer lines of solar origin which are for the most part considerably narrower than those of his second paper. For the second order (which

[1] *Astrophysical Journal*, **69**, 322, 1929.

he used) at λ 4550, β is 17.6 milliangström, so that some of his lines are distorted as much as our Figure 2 indicates.[1] The further results of Woolley's paper No. 1 are therefore very inaccurate[2] and practically useless.

As M. Minnaert and G. F. W. Mulders[3] have found from measures of total absorption (which is independent of resolving power), there is a real effect of the kind noted by Woolley which is due to thermal Doppler effect, but we shall show that this effect can never be inferred from actual contour measures with gratings of the usual size, but only by the method of Minnaert and Mulders who measured total absorptions, or by the use of interferometers.

III. SOME NOTES CONCERNING WOOLLEY'S THIRD PAPER

Doppler effect.—We proceed to supplement Woolley's treatment of Doppler effect by proving the statement given at the end of section II. The Doppler widening of a spectral line is given by

$$I \backsim e^{-\left(\frac{\Delta\lambda}{\gamma}\right)^2},$$

where $\gamma = \lambda/c\sqrt{2kT/m_H M}$ (k = Boltzmann constant, c = velocity of light, m_H = mass of H − atom, M = atomic weight). For $\lambda = 4550$ A and $T = 5000°$ we get $\gamma = 138/\sqrt{M}$ milliangström, which gives for $Fe\ \gamma = 18.5$. This is almost exactly equal to the β for the second order of the 75-foot spectrograph at λ 4550, so that we can say that line-contours measured by a grating of the usual size become meaningless at about the same line-width where Doppler effect becomes predominant.

[1] It should be noted that higher orders usually do not give the theoretical resolving power.

[2] In this connection I notice that among my former measurements (*Zeitschrift für Physik*, **46**, 765, 1927) there is one line which must be affected by instrumental widening, namely, *Sr* 4607. The *NH* given there is something like two or three times too large. This brings the electron pressure computed from Ca^+/Ca and Sr^+/Sr into better agreement. The instrumental test of using two neighboring orders of the grating (*ibid.*, p. 770), also used by Woolley (No. 3, p. 195), is apparently insufficient, because the actual gain in resolving power is—particularly for absorption lines—not the expected one. Only a change by two orders (A. Unsöld, *Astrophysical Journal*, **69**, 322, 1929) gives a clear effect.

[3] *Zeitschrift für Astrophysik*, **1**, 192, 1930 and **2**, 165, 1931.

Russell-Adams calibration.—Since the third paper of Woolley went to press, the question of the limitations of the Russell-Adams calibration has been solved, chiefly by a paper of Minnaert and Mulders.[1] In order to clarify Woolley's considerations it may be useful to state here the solution of the problem: For strong lines the calibration gives the number of atoms NH multiplied by an atomic factor which I called[2] f' (not the usual oscillatory power). For weaker lines (Rowland intensity < 5) the Doppler effect becomes important; the calibration should differ for elements of different atomic weight M. Finally, for very weak lines the total absorption is determined solely by NHf; therefore, if we take NHf' as one co-ordinate, every calibration curve for a certain M splits up into a number of curves according to the ratios f'/f. The differences between the various curves are not so large, however, that the Russell-Adams curve could not be used for many purposes as a first approximation.

Exchange of radiation; interlocking of absorption lines.—Several astronomers have questioned the assumption underlying the current theory of Fraunhofer lines, that a light quantum absorbed in some frequency near the line is re-emitted in exactly the same frequency ("scattering").

For resonance lines this assumption has been proved by the quantum-mechanical calculations of V. Weisskopf,[3] so that doubting it means doubting the whole quantum mechanics.

For penultimate lines the problem is much more difficult. There interlocking can take place and if, for instance, we take an atom with the consecutive states 1, 2, ... , and let the electron jump $1 \to 2$, $2 \to 3$ . . . . $5 \to 2$, $2 \to 1$, the quantum emitted at the last transition will generally differ from that absorbed in the first one by a $\Delta\lambda$ of the order of the line-widths. If such processes become predominant, the whole re-emission will approach the character of thermal emission $\sim \kappa B$, where B should be nearly the Planck function. So we can with a high degree of approximation describe every line by superposing scattering and absorption, where the re-emission corresponding to the latter need not have exactly the equilibrium value.

More information cannot be obtained from general theoretical

[1] *Ibid.*, **2**, 165, 1931.

[2] *Ibid.*, p. 199, 1931.

[3] *Annalen der Physik*, 5. Folge, **9**, 23, 1931.

considerations. But in the case of the sun, observation allows further conclusions: The fact that most of the Fraunhofer lines change very little from center to limb indicates, according to Schwarzschild's classical paper, that the re-emission has predominantly the character of scattering, so that the application of the usual theory (line-width$\sim\sqrt{NHf'}$) must give very nearly correct results.

The "absorption" produced by interlocking gives a contribution to the central intensity.[1] Considering the results of A. Pannekoek[2] concerning the collision theory of central intensities,[3] we may ask whether this process does not give the whole central intensity.[4] With this assumption all the conclusions of the former theory would remain valid except the dependence on pressure. It will be an interesting problem for observational astronomers to find out whether the dependence of central intensity on absolute magnitude[5] is not perhaps a purely instrumental effect due to different line-widths combined with lack of resolving power.

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[1] Cf. also S. Rosseland, *Astrophysik*, p. 144, Berlin, 1931.

[2] *Monthly Notices of the Royal Astronomical Society*, **91**, 139 and 519, 1930–1931.

[3] A. Unsöld, *Sommerfeld-Festschrift*, p. 95, Leipzig, 1928.

[4] Also the so-called "resonance" lines, like *NaD*, Ca^+ H-K, have some interlocking in their upper quantum states.

[5] C. H. Payne, *Harvard Circular*, No. 306, 1927.

THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG STARS BRIGHTER THAN THE SIXTH APPARENT MAGNITUDE[1]

By GUSTAF STRÖMBERG

ABSTRACT

1. The distribution of *visual absolute magnitudes* among stars of spectral types B, A, F, G, K, and M brighter than the sixth visual apparent magnitude is given in Table I. Variable stars and faint companions of binaries have been excluded from the analysis. The methods used in deriving the absolute-magnitude distribution are outlined in *Mt. Wilson Contr.* Nos. 395 and 410. The distribution, when all spectral types are combined, is also given in Table I. The same data are shown graphically in Fig. 1, in the form of twelve distribution-curves all reduced to equal numbers of stars.

2. Data are given by which the visual absolute magnitude can be reduced to *bolometric absolute magnitude*, or expressed in *total power output*. Fig. 2 shows the distribution-curves expressed in bolometric absolute magnitudes.

3. Evidence is given for the existence of five *chains* or *sequences* connecting the different values of the absolute magnitude for which the frequencies have maximum values. These five chains are denoted by the terms *main sequence, normal giants, faint giants, bright giants*, and *supergiants*. Two *gaps* exist in the diagrams: the well-known gap separating giants from dwarfs among the later types; the other separating the normal giants or the main sequence stars from the bright giants and extending from spectral type B3 to about G8. The latter gap represents possibly a region of instability connected with Cepheid variation and outbursts of novae.

1. Determinations of the distribution of absolute magnitude among stars brighter than the sixth apparent visual magnitude for eleven subdivisions of the spectral interval Bo–M have been published in *Mount Wilson Contributions* Nos. 411,[2] 418,[3] 430,[4] and 440.[5] The methods of derivation, outlined in *Contributions* Nos. 395[6] and 410,[7] are based on the observed distributions of reduced angular parallactic and peculiar motions and of radial velocities. A summary of the results is given in this paper, and attention is called to some features which become apparent when the results are brought together.

2. Individual stars equal to or brighter than apparent visual magnitude 6.0 and classified as B, A, F, G, K, or M have been included. Of binaries the brighter star has been used, if above the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 442.

[2] *Astrophysical Journal*, **72**, 117, 1930.

[3] *Ibid.*, **73**, 40, 1931.

[4] *Ibid.*, **74**, 110, 1931.

[5] *Ibid.*, **74**, 342, 1931.

[6] *Ibid.*, **71**, 163, 1930.

[7] *Ibid.*, **72**, 111, 1930.

specified limit of brightness. Variable stars were in general excluded, although a few having a very small range of variation were included. Known spectroscopic binaries were included; whenever possible, the magnitude of the brighter component was estimated from the difference in brightness of the spectra. Stars of peculiar spectra, in particular those having bright or sharp lines, were all included. Although Mount Wilson spectra for later-type stars have been used to some extent, the results may be regarded as referring to the H.D. classification.

3. A summary of the distribution of visual absolute magnitudes (unit of distance = 10 parsecs) is given in Table I for the eleven spectral subdivisions and also for all types taken together. The third line of the heading indicates the estimated spectral intervals in a continuous decimal system. The total number of stars and the mean absolute magnitude for each interval are at the bottom of the table. The data are from the previous publications, with actual numbers of stars now introduced. The total for all spectral types is 4192 stars. One B5 star of absolute magnitude -6.5 has been added in order to localize a certain maximum, the existence of which is indicated in Figure 9 of *Contribution* No. 440.

4. All the luminosity-curves, reduced to a total of 1000 stars for each curve, are brought together in Figure 1. A study of this diagram is of great interest and reveals the existence of the two known sequences, the "main sequence," or "dwarf branch," and the "normal giants," and suggests the possible existence of three other sequences.

Starting with the *main sequence*, we see first that there are no dwarf stars of class M brighter than apparent magnitude 6. The dwarf K stars are well separated from the giants. Among the G stars are many real dwarfs, mixed, however, with a new group, which appears also among the K stars and in previous publications is designated by the term "faint giants." The maximum corresponding to the main sequence is clearly marked by a group of faint F stars, which may well be a mixture of dwarfs and faint giants. In spectral type A, the main sequence stars and the normal giants merge into a combined group having a larger dispersion than that for most of the other groups. The main sequence can be traced on through the

TABLE I

DISTRIBUTION OF VISUAL ABSOLUTE MAGNITUDES AMONG STARS BRIGHTER THAN THE SIXTH VISUAL APPARENT MAGNITUDE

M	H.D. SPECTRUM, ASSUMED SPECTRAL RANGE, AND MEAN SPECTRUM											
	B0–B2 B0–B2 B1	B3 B3–B4 B3	B5 B4–B6 B5	B8–B9 B7–B9 B8	A0 A0–A1 A0	A2–A5 A1–A7 A4	F0–F8 A8–F9 F3	G0–G5 F9–G7 G3	K0–K2 G8–K3 K1	K5 K4–K7 K5	Ma–Md K8–M9 M1	B0–Md B0–M9
−8.6	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−8.4	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−8.2	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−8.0	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−7.8	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−7.6	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
−7.4	0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
−7.2	0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
−7.0	0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
−6.8	0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
−6.6	0	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	1
−6.4	0	1	0	.....	1	.....	.....	.....	.....	.....	.....	2
−6.2	0	1	0	.....	1	.....	.....	.....	.....	.....	.....	2
−6.0	0	1	0	.....	2	.....	.....	.....	.....	.....	.....	3
−5.8	1	1	0	.....	2	.....	.....	.....	.....	.....	.....	4
−5.6	1	0	0	.....	2	.....	.....	.....	.....	1	.....	4
−5.4	1	0	0	.....	2	.....	1	.....	.....	1	1	6
−5.2	0	0	0	.....	2	.....	1	1	.....	1	1	6
−5.0	0	1	0	.....	1	.....	1	2	.....	1	2	8
−4.8	1	3	1	1	1	1	1	2	.....	1	2	14
−4.6	1	5	2	2	1	2	1	2	.....	1	3	20
−4.4	3	6	3	4	0	3	2	3	2	1	3	30
−4.2	4	6	4	5	0	4	3	4	3	1	2	36
−4.0	6	7	4	6	0	6	4	5	5	1	2	46
−3.8	6	7	4	7	0	7	5	5	6	1	1	49
−3.6	8	6	4	7	0	8	5	6	8	1	0	53
−3.4	12	4	3	6	0	9	4	6	9	1	0	54
−3.2	17	0	1	5	0	9	4	5	11	1	0	53
−3.0	18	0	0	5	0	8	3	4	12	1	0	51
−2.8	14	0	0	3	0	8	4	5	13	1	0	48
−2.6	8	1	0	2	0	7	4	6	13	2	0	43
−2.4	5	3	1	0	0	5	4	7	13	2	0	40
−2.2	3	10	5	0	0	3	2	7	12	3	1	46
−2.0	2	20	11	0	0	2	0	4	11	3	3	56
−1.8	1	26	16	0	0	1	0	1	9	4	6	64
−1.6	1	30	18	0	0	0	0	0	7	5	8	69
−1.4	0	30	18	1	1	0	0	0	5	5	10	70
−1.2	0	26	15	7	1	0	0	0	5	7	12	73
−1.0	1	17	11	31	2	0	0	0	7	9	13	91
−0.8	1	14	8	42	3	0	0	0	16	12	14	110
−0.6	1	10	7	44	5	0	0	0	33	14	15	129
−0.4	1	6	6	44	9	1	1	0	69	17	15	169
−0.2	1	3	4	41	24	4	4	4	103	18	15	221
0.0	.....	1	2	31	45	15	9	32	139	16	15	305

TABLE I—*Continued*

M	H.D. Spectrum, Assumed Spectral Range, and Mean Spectrum											
	Bo–B2 Bo–B2 B1	B3 B3–B4 B3	B5 B4–B6 B5	B8–B9 B7–B9 B8	Ao Ao–A1 Ao	A2–A5 A1–A7 A4	Fo–F8 A8–F9 F3	Go–G5 F9–G7 G3	Ko–K2 G8–K3 K1	K5 K4–K7 K5	Ma–Md K8–M9 M1	Bo–Md Bo–M9
+0.2	.....	.....	1	22	52	29	13	48	141	12	14	332
+0.4	.....	.....	1	13	54	45	17	49	136	9	12	336
+0.6	.....	.....	1	9	54	52	22	35	86	6	7	272
+0.8	.....	.....	.....	6	50	54	27	20	29	5	6	197
+1.0	.....	.....	.....	4	40	54	31	9	13	4	3	158
+1.2	.....	.....	.....	2	28	53	34	3	11	3	1	135
+1.4	.....	.....	.....	1	13	49	34	2	9	3	1	112
+1.6	.....	.....	.....	.....	9	40	26	2	8	2	.....	87
+1.8	.....	.....	.....	.....	6	32	16	5	7	2	.....	68
+2.0	.....	.....	.....	.....	3	22	9	8	7	1	.....	50
+2.2	.....	.....	.....	.....	2	10	8	9	7	1	.....	37
+2.4	.....	.....	.....	.....	.....	6	13	10	8	1	.....	38
+2.6	.....	.....	.....	.....	.....	3	22	11	9	0	.....	45
+2.8	.....	.....	.....	.....	.....	2	33	11	9	0	.....	55
+3.0	.....	.....	.....	.....	.....	2	38	11	8	0	.....	59
+3.2	.....	.....	.....	.....	.....	1	38	10	6	0	.....	55
+3.4	.....	.....	.....	.....	.....	1	35	8	3	0	.....	47
+3.6	.....	.....	.....	.....	.....	.....	25	7	1	0	.....	33
+3.8	.....	.....	.....	.....	.....	.....	17	7	0	0	.....	24
+4.0	.....	.....	.....	.....	.....	.....	11	7	0	0	.....	18
+4.2	.....	.....	.....	.....	.....	.....	7	7	0	0	.....	14
+4.4	.....	.....	.....	.....	.....	.....	4	6	0	0	.....	10
+4.6	.....	.....	.....	.....	.....	.....	1	5	0	0	.....	6
+4.8	.....	.....	.....	.....	.....	.....	.....	2	0	0	.....	2
+5.0	.....	.....	.....	.....	.....	.....	.....	1	0	0	.....	1
+5.2	.....	.....	.....	.....	.....	.....	.....	1	1	0	.....	2
+5.4	.....	.....	.....	.....	.....	.....	.....	.....	1	0	.....	1
+5.6	.....	.....	.....	.....	.....	.....	.....	.....	2	0	.....	2
+5.8	.....	.....	.....	.....	.....	.....	.....	.....	2	0	.....	2
+6.0	.....	.....	.....	.....	.....	.....	.....	.....	2	0	.....	2
+6.2	.....	.....	.....	.....	.....	.....	.....	.....	2	1	.....	3
+6.4	.....	.....	.....	.....	.....	.....	.....	.....	2	1	.....	3
+6.6	.....	.....	.....	.....	.....	.....	.....	.....	1	1	.....	2
+6.8	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	1
+7.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	1
+7.2	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Total..	124	246	152	351	416	558	544	405	1022	186	188	4192
$\overline{M}$....	−3.26	−1.86	−1.63	−0.74	+0.42	+0.53	+1.74	+0.69	+0.08	−0.40	−0.77	+0.05

series of B stars to a maximum luminosity of $M = -2.9$ for the early B's. The change in absolute magnitude from B5 to B3 is 0, but from B3 to B1 amounts to 1.5 mag.

The maxima corresponding to the *normal giants* can be followed from the normal giant M stars through the types K, G, and F, the

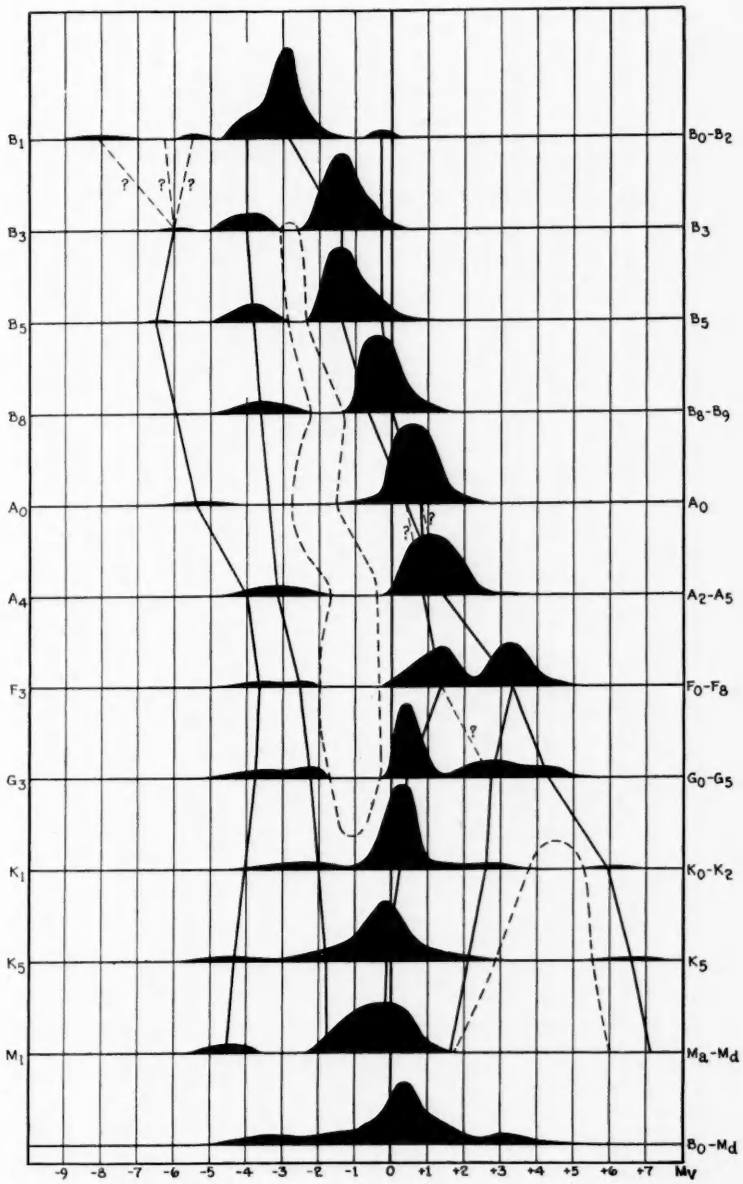


FIG. 1.—Distribution of visual absolute magnitudes among stars brighter than the sixth apparent magnitude.

luminosity gradually decreasing. Among the A and late B stars the normal giants increase in brightness and are mixed with the main sequence stars. An asymmetry for the intermediate B's and an isolated group for B0–B2 indicate that the main sequence branch and the line of normal giants may really have crossed among the A stars. We may, however, also regard the bulk of B stars as consisting of normal giants instead of main sequence stars. On this assumption there is no crossing of the branches. These alternative connections are marked in the figures by dash-lines with question marks.

As for evidence of other sequences, attention has already been called to the group of *faint giants* branching off at the F's and running parallel to the line of normal giants from F to K, and perhaps even to M. Further, groups of very bright stars appear to the left of the normal giants and in many cases are well separated from them. The known c stars and pseudo-Cepheids must be included in these groups, but, judged from their relative numbers, other stars must also be involved. An appreciable number of pseudo-Cepheids brighter than apparent magnitude 6 are known to exist among the late K stars. A group of stars considerably brighter than the normal giants was found among the early K stars, but these cannot be c stars or pseudo-Cepheids, since only three such objects are known among the numerous stars of type K0–K2. This group of *bright giants*, which is regarded as distinct from the supergiants, has been found to exist among the stars of all spectral types except A0. Even if they sometimes appear to merge with the supergiants, their existence is always indicated by the relative abundance of stars brighter than the normal giants, which, in the earlier types at least, cannot be accounted for by the relatively small number of known c stars. Among the earliest B stars their existence is indicated by a bulge on the principal group.

The *supergiants* are prominent among the M, K5, G, F, and A0 stars, but for some reason do not appear in the group B8–B9, although this spectral interval includes seven known c stars brighter than apparent magnitude 6. For this part of the luminosity-curve, however, we can hardly expect high accuracy, particularly when the number of stars is as small as it is in the case of the B's. Thus, as

pointed out in *Contribution* No. 440, we cannot depend on the reality of the groups of extremely high luminosity for the B0–B2 stars. The remarkable thing is, rather, that consistent results can be obtained at all for these stars of very small proper motion.

Although the existence of bright giants and supergiants as well-separated groups can hardly be regarded as proved, the *clear gap* between normal giants and main sequence stars on the one hand and supergiants and bright giants on the other is fairly well established for spectral types B3–G, and perhaps also among the M stars. This gap in the distribution is marked by a closed dash-line in Figure 1. It may perhaps be regarded as an extension of the so-called "Hertzsprung gap," which was introduced to describe the deficiency of stars of spectral type A having an absolute magnitude brighter than zero. It is difficult to establish the existence of this new gap, referring as it does to stars of very high luminosities, by the use of trigonometric parallaxes, since the large percentage errors produce a very great additional spread; but perhaps this may be done by studies of star clusters and of the Magellanic clouds. The gap may perhaps serve as a means of determining the zero-point of the luminosity-curve for star clusters, and hence their distances.

The Cepheid variables, which were excluded from this analysis, have an absolute magnitude ranging from about 0 for the cluster type up to about -5 for those with the longer periods. For types F and G the long-period Cepheids have a mean visual absolute magnitude of about -1.5 and would mainly fall within the gap just mentioned. It may be that the low frequency of non-variable stars in this region of absolute magnitudes is connected with the form of instability exhibited by the Cepheids, the cluster-type variables falling at the right-hand boundary of the gap. Among the A and B stars there is a region of absolute magnitude within which not even variables seem to exist, and which may correspond to the instability of a star in the pre-nova state.

The other gap, also shown in Figure 1 by a dash-line, corresponds to the well-known deficiency of stars of later types intermediate in luminosity between giants and dwarfs.

Table II gives the numerical data for the sequences just described. The figures must not be accepted too confidently, particularly those

referring to the brightest stars and to groups which are assumed to be mixtures of two groups having different luminosities.

The foregoing results may be reduced to photographic absolute magnitudes by adding to the visual absolute magnitudes the color-indices for the different spectral types.[1]

TABLE II

MEAN VISUAL ABSOLUTE MAGNITUDES

Harvard Spectrum	Mean Spectrum	No.	Super-giants	Bright Giants	Normal Giants	Main Sequence	Faint Giants	All
B0–B2..	B1	124	−8.? −6? −5.5?	−4.0:	−0.3	−2.9	.....	−3.3
B3.....	B3	246	−6.0	−4.0	−0.3:	−1.4	.....	−1.9
B5.....	B5	152	−6.5?	−3.8	−0.3:	−1.4	.....	−1.6
B8–B9..	B8	351	?	−3.6	−0.4 0.0:	−0.7:	.....	−0.7
A0.....	A0	416	−5.4	.....	+0.6 +0.8:	+0.4:	.....	+0.4
A2–A5..	A4	558	−3.2 −4.0	−3.1	+1.1 +0.8:	+1.4	.....	+0.5
F0–F8...	F3	544	−3.6:	−2.5:	+1.4	+3.3 +3.3	+3.3	+1.7
G0–G5..	G3	405	−3.7:	−2.2:	+0.4	+4.3	+2.7:	+0.7
K0–K2..	K0	1022	−4.1	−2.0:	+0.2	+6.0	+2.6:	+0.1
K5.....	K5	186	−4.3	−1.8:	−0.1	+6.6	+2.1?	−0.4
Ma–Md.	M1	188	−4.6	−1.6?	−0.2	.....	+1.6?	−0.8

5. More important physically than the visual or photographic magnitude of a star is its bolometric magnitude, which expresses the total radiation of the star as measured outside the earth's atmosphere. To convert the visual absolute magnitudes into bolometric values we may use the formula of Pettit and Nicholson,[2]

$$M_b = M_v - \text{H.I.} - \Delta m_r + 0.9 ,$$

where M_b is the bolometric absolute magnitude, M_v the visual absolute magnitude, H.I. the heat-index, and Δm_r the correction for atmospheric extinction. The zero-point for bolometric magnitudes is so determined that for eight giant F5–G5 stars the mean visual

[1] F. H. Seares, *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 165, 1922, Table XII.

[2] *Mt. Wilson Contr.*, No. 369, eq. (6); *Astrophysical Journal*, **68**, 279, 1928.

magnitude, reduced to the zenith, equals the mean bolometric magnitude.

Table III gives the necessary data[1] and the resulting corrections, $M_b - M_v$. The results for the giants may be regarded as also valid for the supergiants and the bright giants.

TABLE III

Mean Sp.	T	Δm_r	H.I.	$M_b - M_v$
B1	21,000°	2.47	0.04	−1.61
B3	18,000	2.10	0.03	−1.23
B5	15,000	1.71	0.01	−0.82
B8	12,000	1.26	0.00	−0.36
A0	11,200	1.16	0.00	−0.26
A4	9000	0.86	0.02	+0.02
gF3	6500	0.54	0.24	+0.12
gG3	5000	0.41	0.58	−0.09
gK1	4000	0.41	0.90	−0.41
gK5	3300	0.46	1.57	−1.13
gM2	3000	0.48	2.2	−1.8
dF3	6500	0.54	0.24	+0.12
dG3	5800	0.47	0.36	+0.07
dK1	5000	0.41	0.55	−0.06
dK5	4400	0.41	1.10	−0.61
dM2	3300	0.46	2.1	−1.7
Sun	5750	0.47	0.46*	−0.03*

* These values may have an error of ±0.1, since the visual and radiometric magnitudes of the sun are uncertain to this extent.

The total power output of a star, or its total intensity of radiation, E, can be obtained from the relation

$$\log E = \log E_s - 0.4\,(M_b - M_s)\,,$$

where $M_s = 4.84$ is the bolometric absolute magnitude of the sun, M_b that of the star, and E_s, the total power output of the sun. For reduction to absolute units we may use the value

$$E_s = 3.79 \times 10^{33} \text{ ergs/sec.},$$

derived from Abbott's measurements.

[1] *Ibid.*, Tables IV and V. The temperatures used are from Russell, Dugan, and Stewart, *Astronomy*. The values of Δm_r are computed from the temperatures given. The heat-indices are observed means for the spectral types. The zero-point for bolometric magnitudes differs from that of Eddington by 0.12 mag., since Eddington assumes $M_b - M_v = 0$ for a star of surface temperature 6500°. On Eddington's scale $M_b - M_v$ is negative or zero.

Within a given spectral type the variation in bolometric magnitude is principally due to a variation in size. Using Stefan's law, we can compute the diameter D of any star from its surface temperature T and its bolometric magnitude M_b. For values relative to the sun the following relation is easily derived:

$$\log \frac{D}{D_s} = 0.2\ (M_s - M_b) - 2 \log \frac{T}{T_s},$$

where D_s and T_s are the diameter and the surface temperature of the sun, respectively.

The eleven distribution-curves, reduced to bolometric magnitude, are given in Figure 2, which is similar in arrangement to Figure 1. Four dotted lines in the figure connect those points on the axes of the abscissae for the distribution-curves that correspond to stars having diameters equal to 1, 10, 100, and 1000 times that of the sun, respectively. The fact that the last line falls within the group of supergiant M's does not necessarily mean that stars having a volume 10^9 times that of the sun really exist. Errors in the distribution-curve and the dispersion in the temperature among the M stars may well account for the calculated result. The supergiant M stars are, however, among the largest stars known, and there is some evidence that Antares may have a diameter at least 500 times that of the sun.

6. It must be remembered that the stars here studied are selected on the basis of apparent visual magnitude. To obtain the distribution of absolute magnitudes in a given volume of space, the relative frequencies given here must be multiplied by factors which increase rapidly with decreasing luminosity. The effect of the selection of apparently bright stars is, first, to increase abnormally the relative percentage of stars of highest luminosity, and, second, to shift the maximum frequency in any group in the direction of higher luminosity. The second effect increases with the dispersion in the group. Because we have selected the brightest of the dwarfs, the mean absolute magnitudes for these stars may be somewhat fainter than the values given in Table II. For groups having small dispersion, on the other hand, the luminosities corresponding to maximum frequencies are little affected by the selection.

7. The data in Figure 2 may be of importance for our knowledge of stellar constitution, and possibly also for stellar evolution. The

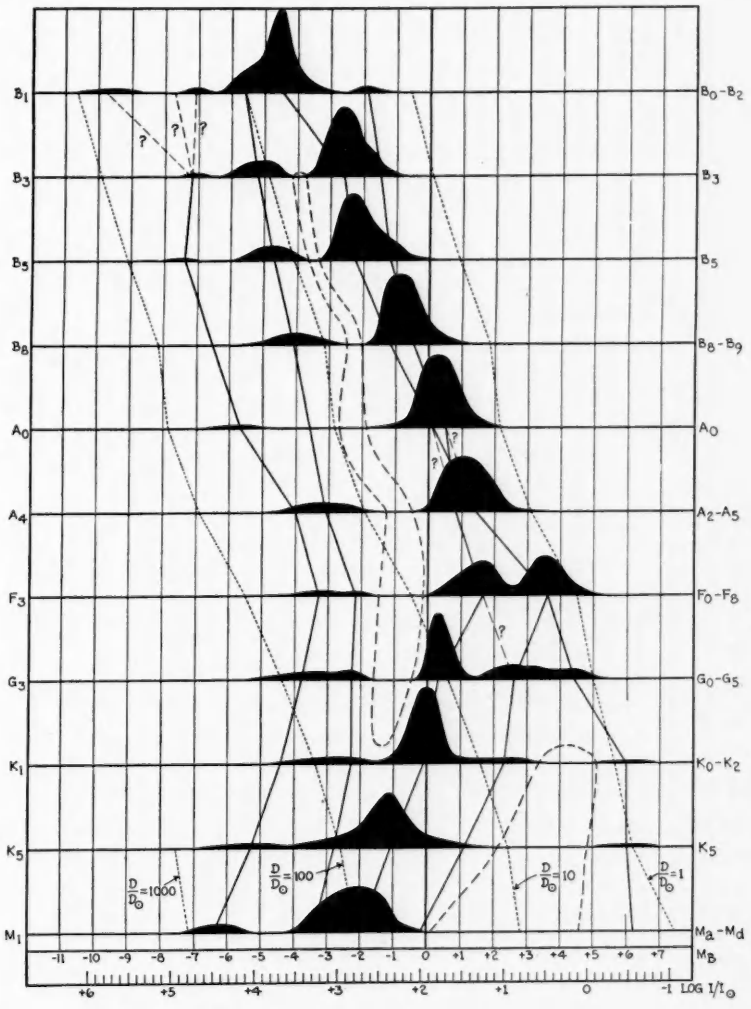


FIG. 2.—Distribution of bolometric absolute magnitudes among stars brighter than the sixth apparent magnitude.

evidence points toward discrete values for the power output at any given surface temperature, and hence toward the existence of different types of stellar structure, with perhaps widely different central

temperatures and different processes involved in the generation of heat.

The evolution, or rather the development, of stars, if such a thing takes place, seems to be clouded in more mystery than ever. The main sequence may be an evolutionary chain, but the other "chains" are all characterized by a minimum of intrinsic brightness at type F and by an increase in brightness among both early and late types—a thing difficult to understand on the basis of uniform evolution. But we must not overlook the possibility that the evolution of a star may not necessarily be accompanied by a continuous change in surface temperature. The stars may have to pass through more or less unstable stages, characterized by pulsations, explosions, or fissions, and then settle down to equilibrium with a surface temperature quite different from that of the previous equilibrium state.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
October 1931

SPECTROSCOPIC OBSERVATIONS OF SX HERCULIS[1]

By ALFRED H. JOY

ABSTRACT

Spectral changes.—Twenty-one spectrograms of SX Herculis were taken at different light-phases. Emission lines of hydrogen reach their greatest strength slightly before maximum and disappear at the time of minimum, giving place to absorption lines. The observed relative intensities are listed in Table II. The spectral type varies from gG4 at maximum to gK0 at minimum. *TiO* bands are superposed on spectra classified as early as G8 and are visible for about 30 days near minimum light. The spectroscopic visual absolute magnitude is -1.5, corresponding to a parallax of $0''.001$.

Radial velocities.—Measures of radial velocity indicate a small variation for both emission and absorption lines. The displacement of the emission lines toward the violet with respect to the absorption lines averages about 4 km/sec.

The variable star SX Herculis (α 16^h3^m3, δ $+25°11'$, 1900; K2, mag. 7.9–9.0) is of especial interest because in many respects it is intermediate between the long-period Me variables and the Cepheid variables. In 1922 W. J. Luyten[2] made a study of spectrograms taken at the Lick Observatory and summarized the light observations made previously to that time.

B. P. Gerasimovič[3] has recently examined the light variations found on the Harvard plates and the visual observations of the American Association of Variable Star Observers. His elements,

$$\text{Max.} = \text{J.D. } 2419608 + 102^d.95E\,,$$

represent the observed maxima with a mean deviation of ± 4.9 days. The period seems to be constant, with small irregularities which may be periodic. He finds a photographic range from 9.3 to 10.5 mag. and a visual range from 7.9 to 9.0 mag. No change in color-index was detected. The time from minimum to maximum is about 40 days.

Twenty-one spectrograms have been obtained at Mount Wilson with the 100-inch telescope and a one-prism spectrograph with 10- and 18-inch cameras. The data for the plates, the observed spectral types, and the measured radial velocities are listed in Table I. The

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 443.

[2] *Lick Observatory Bulletin*, **11**, 124, 1924.

[3] *Harvard Bulletin*, No. 869, 1929.

phases have been calculated from the elements of Gerasimovič. An "e" following the symbol for spectral type indicates the presence of emission lines of hydrogen. In case the titanium bands appear, the classification according to the strength of the bands is given in parentheses. The weights assigned to the measures of radial velocity have been arbitrarily chosen and are based on the quality of the

TABLE I

OBSERVATIONS OF SX HERCULIS

PLATE	DATE	J.D.	SPECTRUM	PHASE	ABS. LINES		EM. LINES	
					Vel.	Wt.	Vel.	Wt.
		2420000+		days	km/sec.		km/sec.	
C 1857	1922 Sept. 3	3301.7	G5e	90.4	+27.4	4	+11.0	3
3953	1926 Aug. 17	4745.7	G4e	93.2	17.6	2	15.8	2
3959	Aug. 19	4747.7	G4e	95.2	22.5	4	12.5	4
4196	1927 March 15	4955.9	G5e	97.4	24.8	2	24.8	2
4219	April 12	4983.9	G6e	22.5	17.3	1	13.6	1
4247	May 8	5009.8	G8(M0)	48.4	18.3	1	.....	....
4404	Sept. 6	5130.7	K0(M3)	66.3	24.9	2	.....	....
4672	1928 Feb. 7	5284.0	G7e	13.8	23.4	2	11.5	1
4736	April 9	4347.0	G9(M1)	76.8	24.7	2	.....	....
4805	May 13	5380.9	G6e	7.7	13.9	2	24.9	2
4840	June 11	5409.7	G7	36.5	10.2	1	.....	....
4858	June 26	5424.7	G7(M0)	51.5	16.4	2	.....	....
4894	July 10	5438.8	G9(M3e)	65.6	21.8	2	8.0	1
γ 16510	1929 April 16	5718.9	G7e	36.8	10.9	1	17.9	2
16574	May 18	5750.7	G8(M2)	68.6	13.9	1	.....	....
C 5212	June 17	5780.7	G3e	98.6	26.8	2	13.4	2
γ 16696	July 13	5806.7	G3e	21.7	14.3	2	11.1	2
C 5389	1930 Feb. 10	6018.0	G6e	27.1	13.6	2	+21.8	2
5409	March 11	6047.0	G8(M1)	56.1	22.9	1	.....	....
5785	1931 July 7	6530.7	G6e	25.0	16.0	2	.....	....
5788	July 8	6531.7	G6e	26.0	+14.5	2	.....	....

spectrum, the dispersion, and the number of lines measured. With the exception of C 1857, 3959, and 4894, which were taken with the 18-inch camera, all the plates were obtained with the 10-inch camera, which gives a dispersion of about 75 A per millimeter at $H\gamma$.

The spectrum is that of a G-type giant. The absolute magnitude at maximum determined from the spectrum with the collaboration of Mr. Adams and Mr. Humason is -1.5, which corresponds to a parallax of $0''.001$. Lines of ionized elements appear with considerable strength, but the c-characteristics are not as marked as in Cepheids of the same spectral type. Throughout most of the period

of the star's variation emission lines of hydrogen are present. As in the Me variables, they are greatly strengthened at maximum light, or shortly before, and disappear at minimum. Estimates of the relative strength of the hydrogen emission lines as actually seen on our plates are detailed in Table II. In general, $H\beta$ is slightly fainter than $H\gamma$, but the series decreases rapidly in intensity in passing

TABLE II

INTENSITIES OF EMISSION LINES

PLATE	SP.	PHASE	INTENSITIES				
			$H\beta$	$H\gamma$	$H\delta$	$H\epsilon$	$H\zeta$
		days					
C 4805	G6e	7.7	20	15	10	2	1
4672	G7e	13.8	10	15	5	0	
γ 16696	G3e	21.7	25	30	30	2	
C 4219	G6e	22.5	25	30	20		
5785	G6e	25.0		2	1	0	
5788	G6e	26.0		2	1	0	
5389	G6e	27.1	25	20	10	1	
4840	G7	36.5	0	0			
γ 16510	G7e	36.8	6	6			
C 4247	M0	48.4	0	0			
4858	M0	51.5	0	a1	a2		
5409	M1	56.1	a1	a2	a2		
4894	M3	65.6	5	2			
4404	M3	66.3	a1	a2			
γ 16574	M2	68.6	0	0			
C 4736	M1	76.8	a1	a1	0		
1857	G5	90.4	70	75	50	2	
3953	G4	93.2	60	70	50	1	1
3959	G4	95.2	30	30	20		
4196	G5	97.4	60	60	25		
5212	G3	98.6	80	80	50	10	2

from $H\gamma$ toward the violet. This behavior is somewhat like that of most of the variables of early Me type having periods less than 250 days and is intermediate between that of the B- and S-type stars, in which $H\beta$ is distinctly brighter than $H\gamma$, and that of the Me variables of late type and longer period, in which $H\delta$ is the strongest line of the series.

At minimum light the absorption lines of hydrogen are present, although much weakened by emission. The strength of these lines is indicated in the table by a figure preceded by the letter "a." The data are not sufficient for a detailed study of the changes in rela-

tive strength of the bright lines during a cycle. It is, however, quite clear that the intensity of the series varies at different maxima and

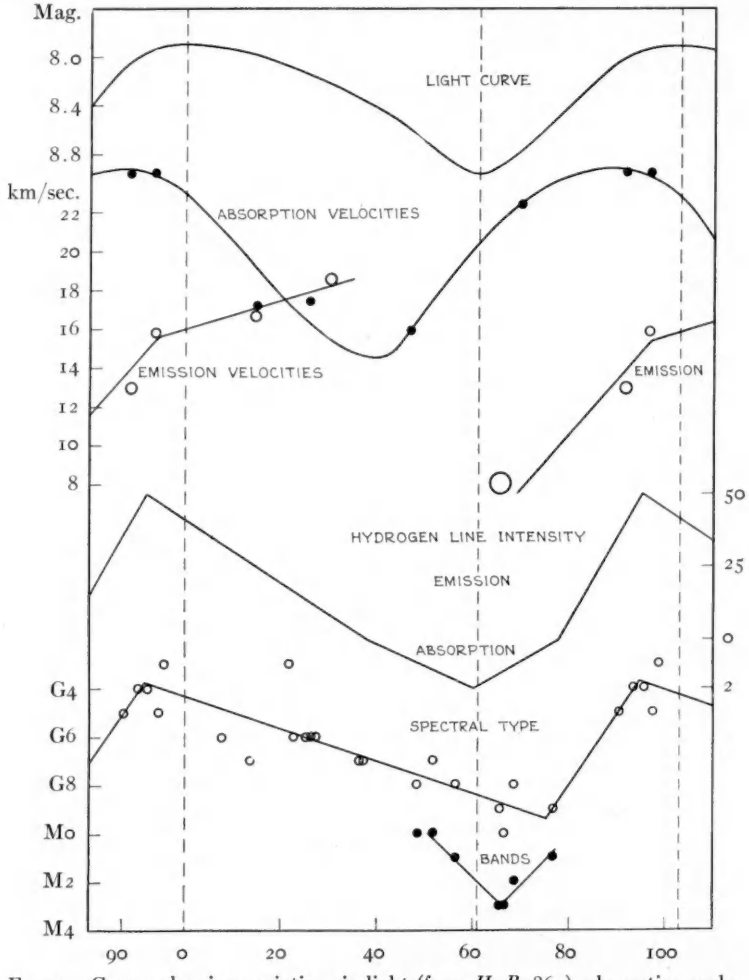


FIG. 1.—Curves showing variations in light (from *H. B.* 869), absorption and emission velocities with normal points, intensities of hydrogen lines in emission and absorption, and spectral type from line criteria and from *TiO* bands.

that the emission is stronger when the spectral type is earlier. Figure 1 pictures in a general way the strength of the hydrogen lines at different phases, in both emission and absorption.

The titanium bands appear in considerable strength near mini-

mum light (Fig. 1, bottom). In only one case out of seven minima are the bright lines present when bands can be seen. The bands begin to appear when the other characteristics of the spectrum indicate type G8. When the bands are strongest the spectrum would be classified as Ko, were the bands not present. This anomalous appearance of the titanium oxide bands has been observed in R Scuti[1] and in several of the RV Tauri variables, but in SX Herculis the bands occur at an earlier stage than in any of the other stars observed. In this respect SX Herculis is quite different from T Centauri (period, 90 days), which has a normal Moe–M4e spectrum. The whole question of the conditions which permit the formation of large numbers of *TiO* molecules in stellar atmospheres in which the temperature of the reversing layer is at least 1000° C. hotter than in normal M-type stars is one which merits more study than has yet been given to it.

The velocity measures of Table I for bright and dark lines have been combined to form the normal places plotted in the upper part of Figure 1. Their weights are nearly the same, except for the emission point at 65.6 days, which is from plate C 4894. The value from this plate is so separated in phase from the others that it could not properly be included in any of the other normal places. It represents the measure of a single line and has low weight. The measures of individual plates show a wide dispersion, but the mean velocity-curve indicates a small variation in velocity during the cycle of the star's light changes that is probably real. Rough curves drawn through the normal places show a range of 9 km/sec. for the absorption lines and 8 km/sec. for the emission lines. At maximum the emission lines are shifted toward the violet relative to the absorption-curve by 10 km/sec., but at a phase of 35 days the corresponding shift is 4 km/sec. toward the red.

The absorption velocity-curve seems to be displaced toward earlier phases by about one-fifth of a period as compared with the light-curve. In Mira Ceti[2] and R Virginis[3] the velocity-curves agree

[1] *Publications of the Astronomical Society of the Pacific*, **34**, 349, 1922.

[2] *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 298, 1926.

[3] Merrill and Joy, *Mt. Wilson Contr.*, No. 382; *Astrophysical Journal*, **69**, 380, 1929.

in phase with the light-curves, while in Cepheids the displacement is so great that opposite phases coincide. The emission-curve of SX Herculis shows little resemblance to the curves of Mira and R Virginis, except that in all three cases the maximum positive velocity shown by the emission lines occurs when the absorption-curves have their minima.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
November 1931

SPECTRUM OF THE IRON STAR XX OPHIUCHI[1]

By PAUL W. MERRILL

ABSTRACT

The *light-variation* is of the R Coronae type. The light-curve since 1919, as determined at Harvard, is shown in Fig. 1.

The *spectrum* is of class Bep. Between 1921 and 1931 it passed through a remarkable *cycle of variation*. At the beginning and end of this period it was marked by emission lines of *H* and *Fe* II, but in 1925 wide, displaced absorption lines appeared, among which lines of *Ti* II were prominent. Estimated *intensities* of bright and dark lines of *H*, *Fe* II, and *Ti* II are in Table II. The behavior of lines of *Cr* II, *Sc* II, *Mg* II, *Al* III, and *He* I is described. The bright metallic lines (*Fe* II, [*Fe* II], *Ti* II, *Cr* II, *Sc* II, *Mg* II, and *Al* III) give *displacements* which agree within observational errors, but the mean displacement of *H* lines is algebraically greater by about 30 km/sec. *Widths of dark lines* were about 6 A in 1925, but decreased thereafter. The *relative displacements of dark components* with respect to the bright lines are shown in Tables V and VI. On plate C 3934 the narrow dark components do not adjoin the bright lines but are displaced toward the violet.

Discussion.—At the beginning and end of the period of observation certain lines were strongest, particularly *H* em and *Fe* II em, while in 1925 the following were strongest: *H* abs, *Fe* II abs, *Ti* em and abs, *Cr* em and abs, *Sc* em and abs. The behavior of a close pair of *Fe* II lines suggests that the absorbing stratum overlay that in which the emission occurred. The positions of the bright lines are nearly independent of the intensities and positions of the dark lines. The great change in spectrum during a period of constant brightness is puzzling. Comparing this star with novae, we may use Halm's expanding-shell hypothesis to explain some of the differences between them by a difference in the relative radii of shell and stellar surface.

This rather faint star[2] was first observed at Mount Wilson because of the Harvard announcements of bright lines in its spectrum. Mrs. W. P. Fleming[3] noticed a well-marked $H\beta$ emission line on an objective-prism photograph taken August 29, 1907. Miss A. J. Cannon,[4] using a plate taken August 4, 1908, classified the spectrum as Bp and remarked, "The lines $H\beta$, $H\gamma$, and $H\delta$ are bright, of which $H\beta$ is the strongest. The dark lines are barely seen, perhaps due to the faintness of the object."

The light variation is of the R Coronae type, normal brightness being photographic magnitude 9.75. The minima are irregular in depth and time of occurrence. The light-curve since 1919, derived from Harvard observations,[5] is in Figure 1.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 444.

[2] H.D. 161114, B.D.−6°4638; R.A. 17^h38^m6; Dec. −6°14′ (1900); variable; max. 9.6; min. 10.7 photographic magnitude.

[3] *Harvard Circulars*, No. 143, 1908.

[4] *Ibid.*, No. 292, 1926.

[5] *Ibid.*; *Harvard Bulletin*, No. 868, 1929; also recent unpublished observations, kindly sent by Dr. Harlow Shapley.

Spectrograms of this object were obtained at Mount Wilson with a single-prism spectrograph attached to the 100-inch reflector every year from 1921 to 1931 except 1924 (Table I). All except five were taken with an 18-inch camera giving a dispersion at $H\gamma$ of 35 A per millimeter; those obtained with shorter cameras are so indicated in Table I.

TABLE I

JOURNAL OF OBSERVATIONS

Plate	Date	J.D.	Remarks
		2420000+	
C 1170	1921 July 23	2894	7-inch camera; poor
1791	1922 July 13	3249	
2341	1923 July 3	3604	
2414	Aug. 31	3663	
3345	1925 June 15	4317	
3350	June 16	4318	
3453	Aug. 11	4374	
3496	Sept. 4	4398	
3698	1926 Feb. 21	4568	10-inch camera
3804	May 28	4664	
3934	Aug. 12	4740	
4235	1927 Apr. 17	4988	10-inch camera
4324	June 23	5055	
4378	Aug. 13	5106	
4714	1928 Mar. 17	5323	
4862	June 27	5425	
5174	1929 Apr. 26	5728	
5238	July 17	5810	
5475	1930 June 11	6139	Underexposed
5549	Sept. 11	6231	10-inch camera
5726	1931 May 8	6470	10-inch camera

NOTES TO TABLE I

C 1170. Seven-inch camera. Poor. Bright lines only. $H\beta$ very strong.

1791. Plate VIII. Numerous sharp bright lines are visible, although the exposure was insufficient to bring out the continuous spectrum.

2341. Some continuous spectrum is present. The superposed bright lines are narrow and well defined. Narrow dark lines are seen on the violet edges of $H\beta$, $H\gamma$, and *Fe* II λ 4924.

2414. Several other bright lines show dark borders.

C 3345, 3350 Plate VIII. A startling change in the aspect of the spectrum caused by the introduction of numerous strong dark lines. Titanium lines conspicuous.

3453. Good exposure. The dark lines are very strong, but narrower than on C 3350. The absorption spectrum now somewhat resembles that of ϵ Aurigae. With the exception of $H\beta$ the bright lines are inconspicuous.

3496. No marked change.

3698. Ten-inch camera. Bright hydrogen lines stronger.

3804. As compared with C 3496, bright $H\beta$, $H\gamma$, and probably bright *Fe* II lines are stronger, while the dark lines are generally weaker.

3934. Plate VIII. The dark lines are narrow and well defined but do not immediately border the bright lines; instead, they are displaced several angstroms toward the violet, a "shoulder" of continuous spectrum lying between the bright and dark components.

4235. Ten-inch camera. Rather poor. Compared with C 3698, the dark lines are relatively weak; the bright lines about the same.

4324. The absorption components are weaker than on C 3934, but a few of the stronger ones are still well marked. They are now back immediately beside the bright components.

4378. Slight changes only.

4714. Underexposed. $H\beta$ absorption much weaker; in fact, not definitely seen. It was well marked on the preceding plate.

4862. $H\beta$ absorption decidedly stronger again; otherwise spectrum much the same.

5174. Plate VIII. Slight changes only. Quite similar to C 2341.

5238. Underexposed. No great change.

5475. Badly underexposed. Several narrow bright lines but no continuous spectrum visible.

5549. Ten-inch camera. Numerous bright lines clearly seen against a background of continuous spectrum.

5726. Ten-inch camera. Continuous spectrum weak; bright lines probably relatively stronger. Forbidden iron lines have increased intensity with respect to the laboratory lines.

During the decade covered by the Mount Wilson observations this unusual spectrum passed through a remarkable cycle of variation. At the beginning and again at the end of the period of observation, the spectrum was marked by numerous emission lines of ionized iron in addition to those of hydrogen, but in 1925 wide, displaced absorption lines made their appearance, among which the lines of titanium were especially prominent.[1] At this time the spectrum was very complicated, and certain regions became congested with bright and dark lines. With study, however, and especially by direct comparison with earlier plates, it became possible to interpret most, although not all, of the details. The notes follow-

[1] *Publications of the Astronomical Society of the Pacific*, **36**, 225, 1924; **38**, 45, 1926.

ing Table I on the general appearance and chief features of each plate give a preliminary idea of the changes of the spectrum and supplement the data in the tables.

INTENSITIES OF LINES

Hydrogen emission lines are present on all plates, but vary in intensity. Except on the first two plates and the last two or three, they are accompanied on their violet edges by dark borders which

TABLE II

ESTIMATED LINE INTENSITIES

Plate	Hydrogen		Iron		Titanium	
	Em	Abs	Em	Abs	Em	Abs
C 1170	15	(0)	10	.....	.....	.....
1791	11	0	10	.....	0	.....
2341	10	5	9	1	1	0
2414	7	5	8	2	1	1
3345	3	16	6	7	4	10
3350	3	16	6	7	4	9
3453	3	14	5	6	4	7
3496	3	12	5	5	4	5
3698	5	11	5	7	3	4
3804	6	9	6	5	2	2
3934	7	8	6	4	2	2
4235	7	7	5	4	2	2
4324	7	8	7	3	3	2
4378	8	6	7	2	3	2
4714	9	2	7	1	2	2
4862	9	4	8	1	1	1
5174	9	2	8	0	1	0
5238	9	(1)	8	0	1	0
5549	7	0	8	0	1	0
5726	11	0	10	0	0	(0)

in 1925 were very wide and strong. Estimates of the intensities of both components, given in Table II, are plotted in Figure 1. A strong inverse correlation between the intensities of bright and dark components is evident.[1] Bright *Hα* is seen on four of the *Hα*-survey plates taken with the 10-inch telescope on May 17 and June 9, 1926, and July 21, 1927 (two plates). It appears to be somewhat more intense on the 1927 plates. Mr. Edwin Hubble has kindly brought

[1] Somewhat similar behavior, although not so well marked, was found in the spectrum of B.D.+11°4673 (*Mt. Wilson Contr.*, No. 381; *Astrophysical Journal*, **69**, 330, 1929).

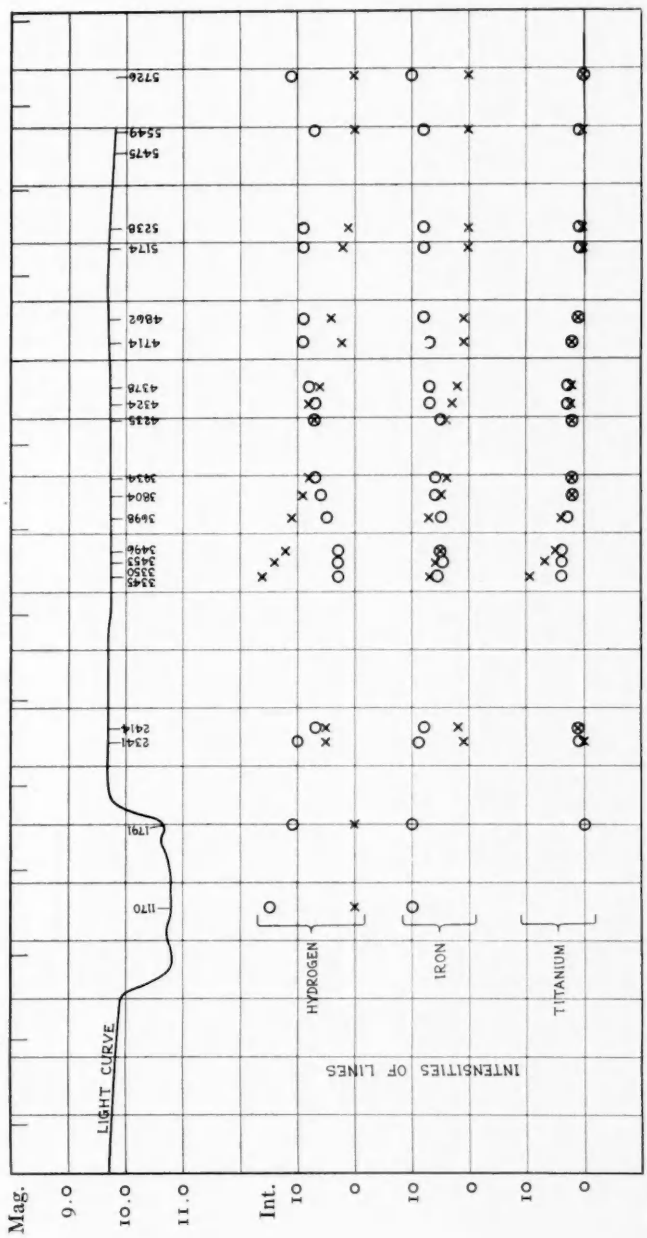


FIG. 1.—Above, light-curve of XX Ophiuchi, showing times at which the spectrograms were taken; below, plots of estimated intensities of spectral lines. Circles represent bright lines; crosses, dark lines.

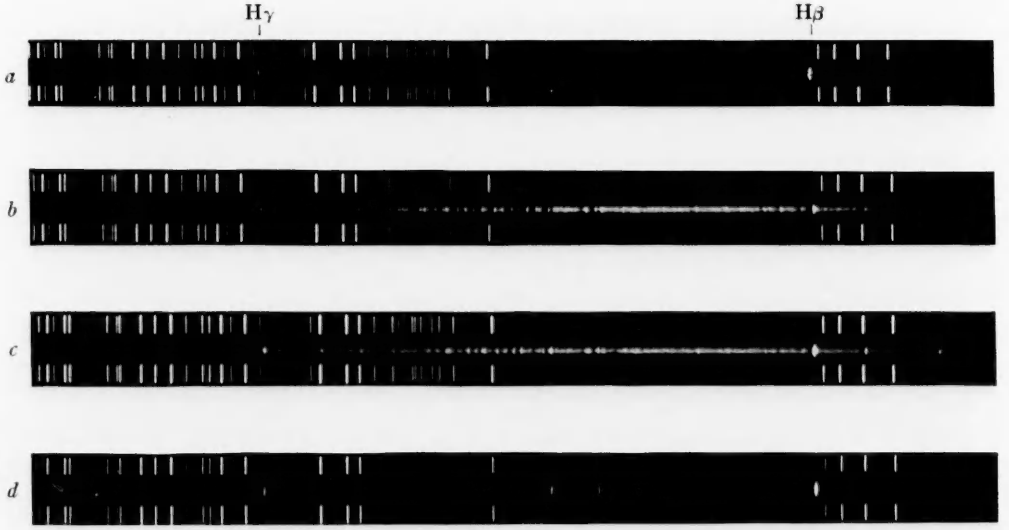
to my attention two objective-prism plates in his collection on which the blue-violet spectrum of this star is recorded. On the first plate, June 21, 1920, the photographic magnitude appears to be about 10.0, while on the second, July 3, 1921, it is considerably fainter, in agreement with the Harvard light-curve. Relative to the continuous spectrum, the bright lines $H\beta$ and $H\gamma$ are probably stronger on the second plate.

Ionized iron emission lines are visible on all the plates, but vary considerably in intensity with respect to the continuous spectrum. Except at the beginning and end of the series of plates, they are accompanied on their violet edges by dark lines of varying intensity (see Table II and Fig. 1). The dark lines are especially strong at λ 4924 and λ 5018, possibly because these lines arise from transitions between levels of small azimuthal quantum numbers. Forbidden lines were measured on eight plates, the lines most frequently observed being λλ 4287, 4358, and 4413 of the $a^6D - a^6S$ multiplet. Except on plates near the beginning and end of the series, they are of very low intensity and are not seen on the 1925 plates. Dark companions were not observed.

Ionized titanium lines, both bright and dark, occur on all plates except a few near the beginning and end of the series (Table II and Fig. 1). During 1925 the dark lines were very wide and strong, dominating the spectrum and giving it a very peculiar appearance (see Plate VIII).

Miscellaneous elements.—Bright lines of ionized chromium are faintly visible on a number of plates, the most prominent being λλ 4558, 4824, and 4848. The dark components of λ 4824 and λ 4848 were well marked in 1925. Ionized scandium emission lines of low intensity have been measured on about a dozen plates, the chief lines being λλ 4246, 4314, 4320, 4324, 4374, and 4670. Absorption components are seen on one or two plates taken in 1925. The general behavior of scandium lines is probably similar to that of titanium and chromium lines. The ionized magnesium line λ 4481 is definitely present in emission on a number of plates, but its intensity seems to vary. A maximum in the spectrum near this wave-length during 1925 may or may not have been due to this line. A well-marked minimum near λ 4479 may be the displaced absorption

PLATE VIII



SPECTROGRAMS OF XX OPHIUCHI

a) C 1791, 1922 July 13. Iron emission predominant
b) C 3350, 1925 June 16. Titanium absorption predominant
c) C 3934, 1926 Aug. 12. Titanium absorption predominant
d) C 5174, 1929 Apr. 26. Iron emission predominant

component. A weak emission line near the position of the ionized aluminum (*Al* III) line λ 4528 was measured on a few plates near the beginning and end of the series. Helium lines are faintly seen in emission on a few plates near the beginning and possibly at the end of the series. On other plates a weak maximum near λ 4471 seems to be the ionized iron line λ 4472.93.

TABLE III

DISPLACEMENTS OF BRIGHT LINES

Plate	*H*	*Fe* II		*Ti* II		*Cr* II		*Sc* II	
	km/sec.	km/sec.	No.	km/sec.	No.	km/sec.	No.	km/sec.	No.
C 1170	(−22.1)	−37.6	9	.....	.....	.....	.....	.....	.....
1791	− 2.1	45.1	23	.....	.....	−51.3	3	(−58.0)	1
2341	−28.7	40.9	28	−41.7	9	−44.0	6	36.4	2
2414	−20.7	36.6	22	17.7	8	(− 8.5)	3	(82.8)	1
3345	+ 5.8	33.7	17	32.2	4	(+ 5.2)	1	(43.4)	1
3350	+ 0.7	25.3	19	29.2	9	−24.3	2	34.9	4
3453	− 2.6	25.7	15	26.2	8	(−22.0)	2	29.2	2
3496	(− 4.1)	33.2	15	33.3	12	−26.0	3	39.0	2
3698	(+ 4.1)	36.3	19	41.9	4	−44.9	2	.....	.....
3804	−13.2	34.5	19	22.5	3	(−46.3)	2	15.7	2
3934	−13.6	42.7	28	33.2	4	(−21.6)	3	31.9	3
4235	(+ 2.6)	26.9	13	(20.4)	1	.....	.....	.....	.....
4324	−13.8	47.8	28	47.3	18	−42.5	7	33.5	4
4378	− 4.2	41.0	28	30.9	13	−23.3	8	24.6	3
4714	(+16.2)	34.2	18	(30.0)	1	−28.1	3	.....	.....
4862	+ 3.1	31.6	27	40.5	10	−22.4	4	(42.6)	3
5174	−11.3	39.3	26	35.4	12	−21.9	3	(15.4)	2
5238	+ 2.4	27.3	18	(40.9)	1	.....	.....	(−32.3)	1
5475	+ 2.9	34.8	10	40.9	2	.....	.....	.....	.....
5549	(− 7.6)	46.1	22	37.2	6	−29.0	3	.....	.....
5726	− 4.8	−44.3	21	(−47.5)	1	.....	.....	.....	.....
Means	− 5.9	−36.4	.....	−34.1	.....	−29.9	.....	−34.8	.....

DISPLACEMENTS OF BRIGHT LINES

The bright lines in this spectrum are narrow, and the stronger ones furnish excellent marks for measurement. The numerous faint lines, although individually not so reliable, in the mean give good results. During 1925 many of the emission lines appeared as bright edges to relatively strong dark lines, but nevertheless they remained well defined and were measurable with reasonable accuracy. All the plates were measured twice by the writer.

The measured displacements of lines of *H*, *Fe* II, *Ti* II, *Cr* II, and *Sc* II on each plate are given in Table III. *Hβ* and *Hγ* were meas-

ured on all the plates, while on eleven plates $H\delta$ was included. For the other elements the number of lines measured is shown in the table. In addition to these measurements the following data are available: From six reasonably accordant plates *Mg* II λ 4481 gave a mean displacement of −43.8 km/sec.; four scattering values of *Al* III λ 4528 yielded a mean of −28 km/sec.; forbidden lines of ionized iron [*Fe* II], measured on eight plates (total of fifteen lines), gave a mean of −36.6 km/sec., in agreement with the result from

TABLE IV

DISPLACEMENTS (KM/SEC.) OF BRIGHT LINES FOR GROUPS OF PLATES

Group	*H*	*Fe* II	[*Fe* II]	*Ti* II	*Cr* II	*Sc* II	*Mg* λ 4481	*Al* λ 4528
I	−17.9	−40.0	.....	−29.7	−39.8	−53.4	.....	.....
II	− 4.7	34.7	.....	32.3	29.1	30.7	.....	.....
II*a*	+ 0.5	29.5	.....	30.2	(18.2)	35.6	.....	.....
III	− 0.6	−36.8	.....	−38.8	−25.4	−30.1	.....	.....
All	− 5.9	−36.4	−36.6	−34.1	−29.9	−34.8	−43.8	−28
H−Others	0.0	+30.5	+30.7	+28.2	+24.0	+28.9	+37.9	+22
+11°4673	0.0	+33.0	+28	+32	.....	.....	.....	+31.5

PLATES INCLUDED IN EACH GROUP

I.....C 1170, 1791, 2341, 2414 (bright-line stage)
II.....C 3345–4378 inclusive (dark-line stage)
II*a*.....C 3345, 3350, 3453, 3496 (extreme dark-line stage)
III.....C 4714–5726 inclusive (recurrent bright-line stage)

the laboratory lines. These and other similar results are given in Table IV for groups of plates chosen to represent different phases of the star's changing spectrum.

WIDTHS OF DARK LINES

In 1925, conspicuous absorption lines gave the spectrum a novel appearance, quite different from that which it had in 1923. A number of the lines were very intense, with well-defined edges, almost as if an occulting bar had been laid across the spectrum during exposure. The strongest lines, due to *H*, *Ti* II, and *Fe* II, extended from their normal positions (marked in most cases by emission lines) several angstroms toward the violet. The greatest widths of the dark lines occurred on plates C3345 and C3350 taken in June, 1925,

on which $H\beta$ measured approximately 6.7 A. The width decreased subsequently as the line became weaker. In June, 1925, the value for the dark $H\gamma$ line appeared to be about 8.8 A, but this abnormal width was probably caused by the extension of the violet edge by the displaced absorption component of *Ti* II λ 4337.92. At this time the widths of iron and titanium lines were about 6 A, while in

TABLE V

DISTANCES IN ANGSTROMS BETWEEN CENTERS OF BRIGHT AND DARK COMPONENTS (INDIVIDUAL LINES)

Plate	$H\gamma$*	$H\beta$	λ 4533 *Fe*(+*Ti*)	λ 4549 *Fe*(+*Ti*)	λ 4583 *Fe*	λ 4924 *Fe*	λ 4501 *Ti*
C 1170	.....	.....	.....	.....	.....	.....	.....
1791	.....	.....	.....	.....	.....	.....	.....
2341	3.27	3.81	.....	.....	.....	3.32	.....
2414	.....	3.66	1.58	1.83	.....	.....	.....
3345	.....	4.22	2.80	3.16	3.55	4.31	3.42
3350	4.96	4.17	3.00	3.35	3.43	3.94	3.39
3453	3.54	3.84	3.06	2.86	2.45	3.28	3.09
3496	.....	3.09	2.73	2.85	2.39	3.36	2.66
3698	3.88	4.74	2.65	3.14	3.28	3.57	2.98
3804	.....	4.40	.....	.....	4.29	4.77	.....
3934	4.68	4.92	4.71	4.85	5.20	4.74	.....
4235	3.30	(5.85)	.....	2.00	2.33	(5.85)	.....
4324	3.15	3.88	1.72	1.88	1.88	2.63	.....
4378	2.86	3.56	1.76	1.77	1.84	2.12	.....
4714	1.74	.....	1.50	1.64	.....	.....	.....
4862	1.98	3.26	.....	.....	.....	.....	.....
5174	2.04	3.71	.....	.....	.....	.....	.....
5238	.....	.....	.....	.....	.....	.....	.....
5475	.....	.....	.....	.....	.....	.....	.....
5549	.....	.....	.....	.....	.....	.....	.....
5726	.....	.....	.....	.....	.....	.....	.....

* On several plates the value for $H\gamma$ may be too large because of the effect of an overlapping line of ionized titanium, λ 4337.92.

August and September of the same year, on plates C 3453 and C 3496, the lines of both elements averaged less than 4 A. In succeeding years the dark metallic lines were weaker and narrower.

DISPLACEMENTS OF DARK LINES

The distances between the effective centers of several bright lines and their dark components are given in Table V. The lines selected are those for which the measurements are most numerous. When the titanium lines were strong, the results for $H\gamma$, as already remarked in connection with the measurements of width, were probably made

too large by the presence of *Ti* II λ 4337.92. On plates C 4862 and C 5174, where the influence of the titanium line appears to be small, the ratio of the separation (emission *minus* absorption) of $H\beta$ to that of $H\gamma$ is about 1.7, whereas the ratio of the wave-lengths is only 1.1. This may be an example of the apparent tendency frequently met in Be stars for the line patterns to increase in size toward the red more rapidly than the wave-length. It happens that

TABLE VI

MEAN DISTANCE BETWEEN CENTERS OF BRIGHT AND DARK COMPONENTS

Plate	*Hβ*	*Fe*		*Ti*		*Cr*	
	A	A	No.	A	No.	A	No.
C 1170	.....	.....	.....	.....	.....	.....	.....
1791	.....	.....	.....	.....	.....	.....	.....
2341	3.81	3.32	1	.....	.....	.....	.....
2414	3.66	1.70	2	2.25	1	.....	.....
3345	4.22	3.51	5	3.63	3	4.48	1
3350	4.17	3.62	12	3.48	9	3.20	1
3453	3.84	3.02	13	2.98	8	3.06	2
3496	3.09	3.00	11	2.83	11	3.32	2
3698	4.74	3.40	16	3.09	3	.....	.....
3804	4.40	4.53	2	.....	.....	4.39	1
3934	4.92	4.48	18	(3.17)	3	5.00	2
4235	(5.85)	2.16	4	.....	.....	.....	.....
4324	3.88	2.12	8	1.38	3	1.32	1
4378	3.56	1.72	9	1.53	4	1.66	2
4714	.....	1.57	2	.....	.....	.....	.....
4862	3.26	.....	.....	.....	.....	.....	.....
5174	3.71	.....	.....	2.32	1	.....	.....
5238	.....	.....	.....	.....	.....	.....	.....
5475	.....	.....	.....	.....	.....	.....	.....
5549	.....	.....	.....	.....	.....	.....	.....
5726	.....	.....	.....	.....	.....	.....	.....

the lines λ 4533 and λ 4549 are blends of iron and titanium lines, but as the distances between the components are only 0.20 and 0.16 A, respectively, the separations of the blended pairs from the centers of the accompanying absorption lines cannot be much greater than for single lines. Independent data (Table VI) show that iron and titanium behave very similarly; hence, the separations measured for these two blends will be about the same regardless of the relative importance of each element. The values for $H\beta$ and the mean values for lines of iron, titanium, and chromium, together with the numbers of lines, are given in Table VI.

The large displacements of the dark lines on plate C 3934 are not caused by increased width or strength of the lines but by some special phenomenon. The lines are narrow and well defined, but, instead of being immediately contiguous to the bright lines, are displaced toward the violet, apparently leaving a narrow strip of continuous spectrum between bright and dark components.

DISCUSSION

In studying the relative behavior of lines of various elements in the spectrum of XX Ophiuchi it is important to notice that the correlation between intensities of bright and dark components is direct for *Ti* II, *Cr* II, and *Sc* II, but inverse for *H* and *Fe* II. With this in mind, we may divide the lines into two groups according to the way their intensities changed. In group I the lines were strongest at the beginning and end of the period of observation, while in group II they were strongest in 1925. Ionized iron lines behave like the hydrogen lines in many Be spectra. The other lines have not often been observed in emission.

Group I	Group II
H em	*H* abs
Fe II em	*Fe* II abs
(*Mg* II em)	*Ti* II em, abs
(*Al* III em)	*Cr* II em, abs
(*He* I em)	*Sc* II em, abs

It is interesting to examine cases in which a dark line was superposed upon one previously bright, for these bear on the problem of the relative levels of absorption and emission. Unfortunately, very few pairs of lines are sufficiently close for this test. The best examples are the iron lines λλ 4520.24, 4522.64 ($\Delta\lambda = 2.40$ A) and λλ 4489.21, 4491.41 ($\Delta\lambda = 2.20$ A). The relative intensity of bright λ 4520 appears decidedly decreased when the dark lines are strong, as if the absorption were in an overlying layer; this is not, however, clearly true of λ 4489. Line λ 4522 may possibly have the greater absorbing power, as it is in the $b^4F' - a^4D'$ multiplet, while λ 4491 is in $b^4F' - a^4F$. On the whole, the facts seem to favor overlying absorption, but the evidence is not convincing.

Table IV shows that the bright lines of *Fe* II, [*Fe* II], *Cr* II, *Sc* II,

Mg II, and *Al* III have practically the same displacements, the differences being perhaps within the errors of determination. The bright hydrogen lines are, however, displaced toward the red by the equivalent of 30 km/sec. (see next to last line of Table IV). This is very nearly the value found in the spectrum of B.D.+11°4673[1] for the displacement of *H* lines relative to those of *Fe* II, [*Fe* II], *Ti* II, and *Al* III (last line of Table IV).

It is obvious from Tables III and IV that the positions of the bright lines are nearly independent of the intensities and positions of the dark lines. A slight displacement toward the red occurred when the absorption lines were strongest (group II*a*, Table IV). The apparent shift of the hydrogen lines in passing from group I to group II or II*a* cannot wholly be ascribed to the effect of the dark lines, however, for practically the same shift persists in group III, in which the dark lines are again weak. The absence of a pronounced effect of the dark lines on the position of the bright lines has also been noted in other peculiar spectra. The phenomenon raises the question whether the dark lines may not be produced in a portion of the atmosphere more or less distinct from that in which the bright lines originate.

The bright iron-line spectrum observed before the curious outbreak of dark lines in 1925 appears to represent a condition of quasi-equilibrium, because after 1925 the spectrum gradually returned to its previous aspect. This is true not only of the general character of the spectrum but also of its details. For example, the spectrum on plate C 5174 taken in 1929 is nearly identical with that on C 2341 taken in 1923.

Relatively faint emission lines of *Ti* II and *Cr* II are associated with *Fe* II emission in several stars besides the one here under consideration, namely, η Carinae,[2] Z Andromedae,[3] B.D.+11°4673,[4] and Z Canis Majoris,[5] the last of which shows spectral changes somewhat resembling those of XX Ophiuchi.

[1] *Mt. Wilson Contr.*, No. 381; *Astrophysical Journal*, **69**, 330, 1929.

[2] *Mt. Wilson Contr.*, No. 354; *Astrophysical Journal*, **67**, 391, 1928.

[3] *Publications of the Dominion Astrophysical Observatory*, **4**, 119, 1928.

[4] *Mt. Wilson Contr.*, No. 381; *Astrophysical Journal*, **69**, 330, 1929.

[5] *Mt. Wilson Contr.*, No. 334; *Astrophysical Journal*, **65**, 286, 1927.

It is remarkable that the increase of one magnitude in the star's brightness from 1922 to 1923 should produce so little effect on the spectrum as that found on comparing plates C 2341 and C 2414 (1923) with C 1791 (1922). A similar phenomenon is known, however, in long-period variables and in novae before maximum, for in these objects a change of one or two magnitudes often occurs without profound modification of the spectrum.

More extraordinary is the marked alteration in the character of the spectrum between 1923 and 1925 quite without any accompanying change in photographic magnitude. The photometric observations in this respect are definite. From the spectroscopic behavior one might infer that the increase in brightness after the flat minimum started a spectroscopic change which continued in a cumulative manner for two or three years, after which occurred a gradual resumption of the earlier conditions.

In 1925 the lines were qualitatively of the P Cygni or nova type and the subsequent weakening and disappearance of the dark lines parallels the history of many novae. The abnormal violet shift of narrow dark lines on plate C 3934 has some analogy in the behavior of certain novae at an early stage. We may therefore inquire whether J. Halm's hypothesis[1] of an expanding shell of emitting and absorbing gas is applicable to this star. At the outset we meet the difficulty that narrow bright lines were present before, during, and after the apparition of the dark lines. Have we, then, one shell for bright lines and another for dark lines? To admit this is to impair seriously the attractive simplicity of Halm's hypothesis. Perhaps a stationary shell composed of outward-moving atoms, as pictured by C. S. Beals to explain Wolf-Rayet spectra,[2] is subject to changes which affect its ability to absorb. It must dispose of the extra energy received by absorption, however, without greatly changing the total bright-line emission.

If we assume that a shell of the Halm type is responsible for both bright and dark lines, a simple geometrical explanation of the different relative widths of bright and dark lines in novae and in XX Ophiuchi presents itself. In novae the bright lines are wide, the dark

[1] *Proceedings of the Royal Society of Edinburgh*, **25**, 513, 1904.

[2] *Publications of the Dominion Astrophysical Observatory*, **4**, 271, 1930.

lines narrow; in XX Ophiuchi (for a time) the reverse was true. This difference might be explained by the relative radii of the stellar photospheric surface and of the emitting and absorping shell (see Fig. 2). A nova has a large shell with only a small fraction of it lying directly between us and the star; its spectrum therefore shows a

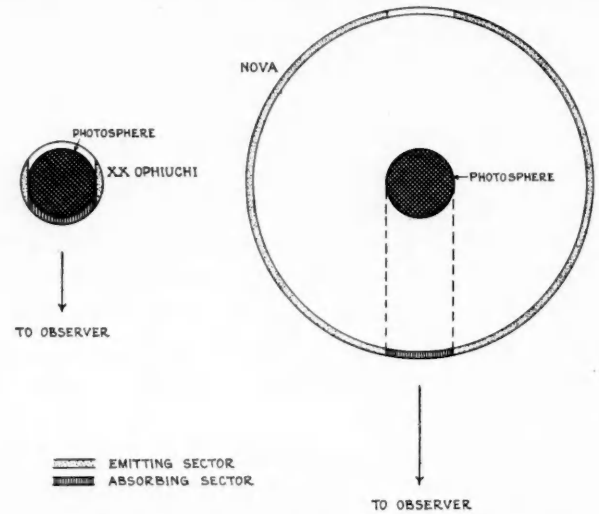


FIG. 2.—Diagram illustrating application to XX Ophiuchi of Halm's hypothesis of novae.

narrow dark line at the violet edge of a broad bright line. In XX Ophiuchi, on the other hand, the spectrum suggests a shell close to the photosphere, a thin ring around the limb emitting a narrow line in its normal position, while the greater part of the expanding shell lies between us and the photosphere and thus produces a broad absorption line. The outward velocity of the atoms would of course be less than that of a nova.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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A SPECTROPHOTOMETRIC STUDY OF Y OPHIUCHI

By C. J. KRIEGER

ABSTRACT

The investigation is based on nineteen spectrograms obtained at the Lick Observatory in 1929. Moll microphotometer tracings of all plates were made.

Spectral type.—The spectral type is found to vary between F5 and G3. The corresponding temperature change, considering Y Ophiuchi as a normal giant, is from 5000° to 6400° K.

Variation of surface temperature.—The intensity distribution of the continuous background derived by Yü's method, with ζ Ophiuchi as a comparison star, gives a blackbody temperature variation from 3500° to 5400°. Mendenhall's method, using B4618 and B4660 as comparison stars, furnishes a temperature variation from 3300° to 5800°. The variation based on color indices is from 3700° to 5700°. The mean variation of the effective surface temperature is from 3600° to 5600°, with a p.e. of an individual determination of $\pm 200°$.

Variation of magnitude.—The magnitude curves at λλ 4200, 4500, and 4800 are derived. The p.e. of an individual determination is $\pm 0^{m}12$.

Line intensities.—The strengths of the β, γ, and δ lines of hydrogen, the H and K lines, and the G band are found in terms of the intensity of the continuous background.

In a previous paper[1] a spectrophotometric study of η Aquilae was made, and it was thought desirable to make a similar investigation of another Cepheid variable, Y Ophiuchi, with regard to the variation both of the surface temperature and of that of the line and band intensities.

I. OBSERVATIONS

During the summer of 1929 a series of observations of Y Ophiuchi was made with the two-prism slitless quartz spectrograph attached to the Crossley reflector at the Lick Observatory (Table I). Eastman-40 plates, developed in D-61, were used throughout.

The observations of Y Ophiuchi were taken with a diaphragm having four circular openings of 10-inch diameter, with the exception of plates 276, 254, and 270, on which Y Ophiuchi was trailed with the full aperture of the telescope. On each plate, excepting 276, 254, and 270, three spectrograms of one comparison star were obtained with the same exposure time as that of the variable, and in an intensity ratio of 1:2:4.[2]

[1] *Astrophysical Journal*, **74**, 10, 1931.

[2] See *ibid.*, Fig. 1.

The positions and characteristics of both the variable star Y Ophiuchi and the comparison stars employed are:

Y OPHIUCHI, H.D. No. 162714, $\alpha\,(1900) = 17^h47^m.3$, $\delta\,(1900) = -6°7'$, $P = 17^d.121$, Max. Phase, J.D. 2425008.80 G.M.T.[1]

ζ OPHIUCHI, Boss No. 4225, H.D. No. 149757, $\alpha\,(1900) = 16^h31^m.7$, $\delta\,(1900) = -10°22'$, Bo, Pm. Mag. 2.70, Pg. Mag. 2.46, $T = 22{,}000°$[2]

Boss No. 4618, H.D. No. 167768, $\alpha\,(1900) = 18^h11^m.6$, $\delta\,(1900) = -3°2'$, gG2, Pm. Mag. 6.11, Pg. Mag. 6.89, $T. = 5200°$[3]

Boss No. 4660, H.D. No. 169689, $\alpha\,(1900) = 18^h20^m.8$, $\delta\,(1900) = +7°59'$, gGo, Pm. Mag. 5.69, Pg. Mag. 6.25, $T = 5600°$[3]

TABLE I

OBSERVATIONS OF Y OPHIUCHI

Plate	Phase	Date 1929	Sid. T.	J.D. 2425000+	Exp. Min.	Comp. Star Boss
274	$0^d.01$	Aug. 23	19^h14^m	847.717	4	4618
276	0.04	Aug. 23	19 58	847.748	3	.....
253	1.12	Aug. 7	18 04	831.712	4	4660
254	1.14	Aug. 7	18 27	831.728	$3\frac{1}{2}$	.....
255	1.14	Aug. 7	18 35	831.734	4	4618
209	2.41	July 5	16 59	798.757	4	4660
210	2.42	July 5	17 14	798.768	4	4618
215	4.38	July 7	16 21	800.726	4	4660
216	4.38	July 7	16 32	800.733	4	4618
231	8.36	July 11	16 08	804.706	4	4660
232	8.36	July 11	16 14	804.710	4	4618
241	10.24	July 30	17 26	823.707	4	4660
242	10.24	July 30	17 32	823.712	4	4618
246	12.23	Aug. 1	17 23	825.701	4	4660
248	12.27	Aug. 1	18 14	825.736	4	4618
268	14.12	Aug. 20	18 46	844.706	4	4618
269	14.13	Aug. 20	18 52	844.710	4	4618
270	14.14	Aug. 20	19 08	844.719	3	.....
272	16.12	Aug. 22	18 55	846.707	$4\frac{1}{2}$	4618

The observations of ζ Ophiuchi used as comparison star in Yü's method for the determination of the distribution of the intensity of the continuous background are listed in Table II.

The observations were taken with a diaphragm having one circular opening of 10-inch diameter, with the exception of plate 252, which was trailed with the full aperture of the telescope.

[1] *Kleine Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg*, No. 5, 1929.

[2] C. S. Yü, *Lick Observatory Bulletin*, **12**, 110, 1926.

[3] *Astrophysical Journal*, **53**, 13, 1921.

II. ESTIMATES OF SPECTRAL TYPES AND TEMPERATURES

The spectral types were estimated on the basis of the Henry Draper system of classification, and temperatures were assigned from the relation established between spectral types and temperatures.[1] For this purpose the Cepheid variables were considered as normal giants, since the table does not give the relations for supergiants. This should be remembered when comparing these temperatures with those obtained by other methods (Table III and Fig. 1*a*). There is good agreement with those derived by Harlow Shapley,[2] also from estimates of spectral types.

TABLE II

OBSERVATIONS OF ζ OPHIUCHI

Plate	Date 1929	Sid. T.	Exp. Min.
208	July 5	16^h01^m	1
235	July 14	16 52	1
240	July 30	16 51	1
245	Aug. 1	16 48	1
251	Aug. 7	17 18	$\frac{1}{2}$
252	Aug. 7	17 28	$1\frac{1}{2}$
267	Aug. 20	18 13	$\frac{1}{2}$
271	Aug. 22	18 16	$\frac{1}{2}$
273	Aug. 23	18 48	$\frac{1}{2}$

III. INTENSITY DISTRIBUTION OF THE CONTINUOUS BACKGROUND AND BLACK-BODY TEMPERATURES

The Moll self-registering microphotometer furnished tracings about 120 mm long, the distance of the lamp and that of the thermocouple from the plate being 200 and 120 mm, respectively. The voltage of the lamp was kept constant at 4.5 volts. Only the central part of the spectrum was measured by attaching a narrow horizontal slit in front of the vertical thermocouple slit, thus avoiding the admission of light falling through the clear plate at the edges of the spectrum. Two sets of sensitometric exposures were impressed on the photographic plate before development, the intensities being in the ratio 1, 2, 4, 8, 16, 32. The first series was made through an ultra-violet filter, Wratten No. 18, with a maximum transmission at λ 3600, and the second series through a blue filter, Wratten No. 50,

[1] Russell, etc., *Astronomy*, **2**, 753, 1927.

[2] *Astrophysical Journal*, **44**, 283, 1916.

with a maximum transmission at λ 4600. The deflection of the galvanometer for clear plate being set equal to 0, and that for complete darkness equal to 100, using the six standard circles for the intermediate values, curves correlating blackening and apparent intensity can be drawn to be used in converting the ordinates of the tracings into apparent intensities. Since a curve is available for the ultraviolet as well as the blue region, a possible change of γ with the wave-length can be taken into account.

TABLE III

ESTIMATES OF SPECTRAL TYPES OF Y OPHIUCHI

(See Fig. 1)

Plate	Phase	Sp. Type	Temp. ° K
274	$0^d.01$	F5	6500
276	0.04	F6	6300
253	1.12	F6	6300
254	1.14	F6	6300
255	1.14	F6	6300
209	2.41	F8	5900
210	2.42	G0	5600
215	4.38	G2	5200
216	4.38	G3	5000
231	8.36	G2	5200
232	8.36	G3	5000
241	10.24	G3	5000
242	10.24	G3	5000
246	12.23	G3	5000
248	12.27	G2	5200
268	14.12	G0	5600
269	14.13	G1	5400
270	14.14	G0	5600
272	16.12	F6	6300

a) Method used by C. S. Yü.—C. S. Yü[1] has described in detail a method for deriving the true (relative) intensities J_λ at any wave-length from the apparent intensities J'_λ furnished by the microphotometer tracings. Briefly,

$$J_\lambda = \frac{J'_\lambda}{a_\lambda (rbds)_\lambda},$$

where a_λ is an atmospheric transmission coefficient, which is a function of humidity, temperature, barometric pressure, altitude, and zenith distance for any wave-length, and is taken from tables;[2]

[1] *Op. cit.*, **12**, 113, 1926.

[2] Smithsonian Physical Tables (7th rev. ed.), pp. 187, 418, 419.

$(rbds)_\lambda$ is an instrument coefficient, depending on the reflectivity of the silver mirror, the transmission and dispersion of the spectrograph, and the sensitivity of the photographic plate, and will be a

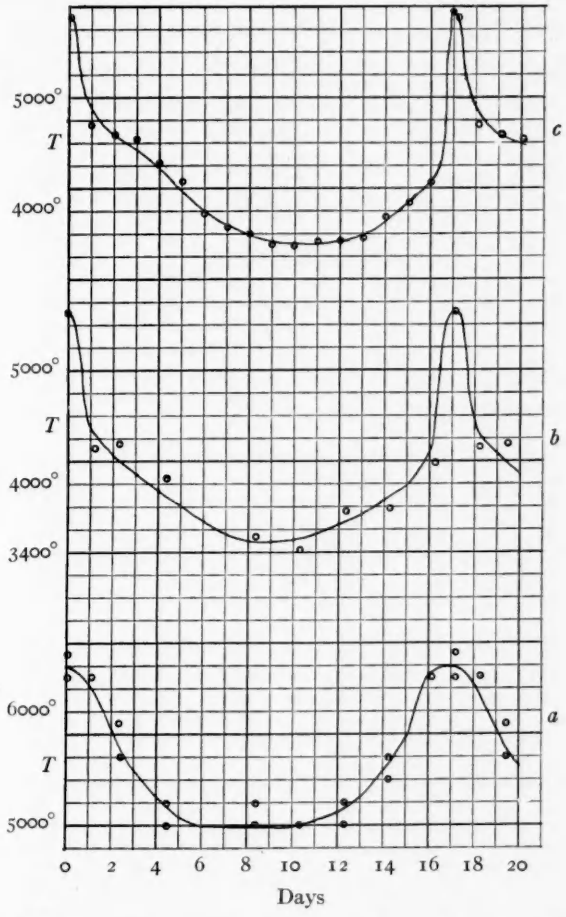


FIG. 1.—(*a*) Temperature variation based on estimates of spectral types; (*b*) black-body temperature based on comparison stars B4618, B4660 and ζ Ophiuchi; (*c*) temperature from color-indices.

constant for any given instrumental arrangement, the same type of plate and development, and a given wave-length. It is determined from a suitable comparison star, assuming that its temperature is known and that it radiates as a black body. ζ Ophiuchi, Bo, 22,000°,

was used as a standard comparison star[1] and the instrument coefficients were derived as shown in Table IV.

Surface 5 was in use from spring, 1928, to July 17, 1929, when a new surface, No. 6, was put on.

The true (relative) intensity distribution of the continuous background was thus derived for the available spectra in the region λ 3400 to λ 5000 at intervals of 200 A, and also at λ 4500, and then expressed in the magnitude scale. The differences of the magnitudes at the wave-lengths λ 4000 and λ 4800, λ 4000 and λ 4600, λ 4200 and λ 4800, λ 4000 and λ 4400, and λ 4200 and λ 4600 were

TABLE IV

INSTRUMENT COEFFICIENT $(rbds)_\lambda$

λ	Surface 5	Surface 6
3400	0.08	0.06
3600	.20	.13
3800	.36	.25
4000	.48	.42
4200	.67	.62
4400	.87	0.92
4500	0.93	1.00
4600	1.00	0.99
4800	0.84	.76
5000	0.33	0.24

formed and the corresponding black-body temperatures computed. The averages are listed in Table V, column 5, under "Temperature Based on Comparison Star ζ Ophiuchi."

b) *Method used by H. S. Mendenhall.*—The method, described by Mendenhall[2] and briefly outlined in a previous paper,[3] for the spectrophotometric observation of variation of magnitude, when three exposures of a comparison spectrum in a known intensity ratio are available, was employed. Using the comparison stars B4618 and B4660, and fitting Planckian curves through wave-lengths λλ 4000, 4200, 4400, 4600, and 4800, the black-body temperatures listed in Table V, columns 3 and 4 ,were derived. The mean of the tempera-

[1] Table II and p. 148 of this article.

[2] *Lick Observatory Bulletin*, **14**, 133, 1930.

[3] *Astrophysical Journal*, **74**, 14, 1931.

tures, based on ζ Ophiuchi, B4618, and B4660, is given in Table V, column 6, and shown in Figure 1*b*.

c) *Variation of magnitude and color-index temperatures.*—The variations of the magnitude at wave-lengths λ 4200, λ 4500 and

TABLE V

BLACK-BODY TEMPERATURE AND MAGNITUDE VARIATION OF Y OPHIUCHI, BASED ON COMPARISON STARS B4618, B4660, AND ζ OPHIUCHI

PHASE	PLATE	TEMPERATURE BASED ON COMPARISON STARS			MEAN TEMP.	MAGNITUDE VARIATION		
		B4618	B4660	ζ Ophiuchi		λ 4200	λ 4500	λ 4800
0.01	274	5780° K	.....	4890	5510	6.77	6.70	6.72
0.04	276	.....	.....	5870				
1.12	253	.....	4420	3590	4320	6.80	6.77	6.53
1.14	254	.....	.....	5170				
1.14	255	4540	.....	3870		7.43	7.34	7.21
2.41	209	.....	4860	3830	4380	7.38	7.18	7.11
2.42	210	4800	.....	3950		7.45	7.25	7.24
4.38	215	.....	3960	4160	4040	7.78	7.59	7.37
4.38	216	3650	.....	4380		7.66	7.29	7.05
8.36	232	3510	.....	3570	3540	7.93	7.49	7.35
10.24	241	.....	3040	3220	3410	8.02	7.53	7.34
10.24	242	3640	.....	3740		7.98	7.73	7.52
12.23	246	.....	3930	3740	3760	7.51	7.31	7.15
12.27	248	3840	.....	3530		7.90	7.64	7.62
14.12	268	4100	.....	3570	3780	7.61	7.44	7.21
14.13	269	3860	.....	3580		7.66	7.41	7.25
16.12	272	3800	.....	4560	4180	7.86	7.61	7.37

λ 4800 are shown in columns 7, 8, and 9, Table V, and are represented graphically in Figure 2*a*, *b*, *c*. From the smoothed curves of variation of the magnitude, columns 2, 3, and 4, Table VI, the differences of magnitude at λ 4200 and λ 4500, λ 4500 and λ 4800, and λ 4200 and λ 4800 were obtained (cols. 5, 6, and 7) and the corresponding temperatures derived (cols. 8, 9, 10, and 11). The average is shown in Figure 1*c*.

The mean of the black-body temperatures (Table V, col. 6) and of the color-index temperatures (Table VI, col. 11) is given in the last column of Table VI, and shown in Figure 2.

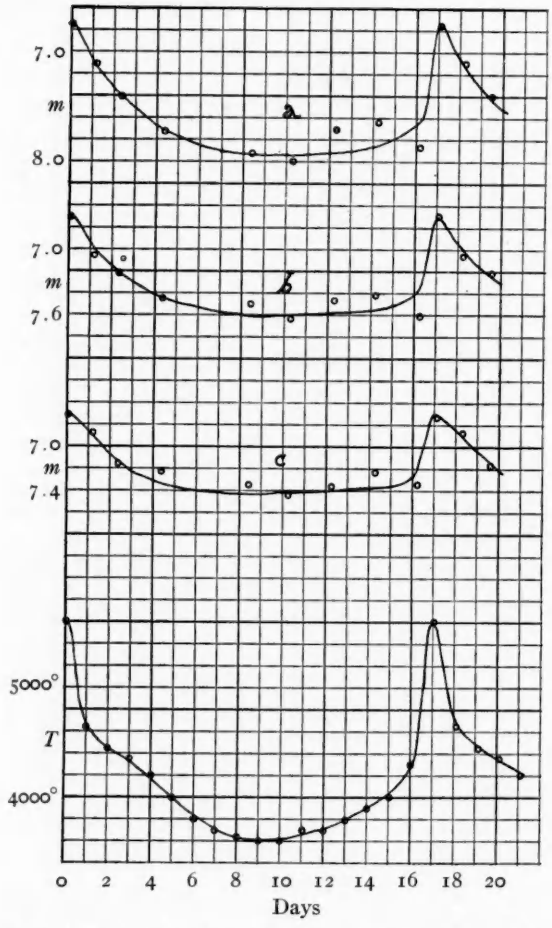


FIG. 2.—Variation in magnitude at (*a*) λ 4200; (*b*) λ 4500; (*c*) λ 4800. At the bottom: mean of the black-body temperatures and of the color-index temperatures.

d) *Discussion.*—A comparison of this curve (Fig. 2) with the temperatures based on spectral-type estimates (Fig. 1) indicates a similar shape, but for Figure 5 temperatures which lie 1000°–1500° below those in Figure 1. Attention has been called[1] to differences in temperature in the descending and ascending branches of the light-

[1] C. H. Payne, *The Stars of High Luminosity*, p. 211, n. 51, 1930.

curve at points of equal brightness for Y Sagittarii,[1] δ Cephei,[2] and Y Ophiuchi.[3] There seems to be an indication that the temperatures in the ascending branch of the light-curve of Y Ophiuchi at λ 4500 are somewhat higher than at points of corresponding brightness on the descending branch (Table VI, cols. 3 and 13). The differences in temperature are very pronounced in η Aquilae.[4] Apparently the same degree of ionization, judging from the estimated spectral types,

TABLE VI

VARIATION OF MAGNITUDE AND TEMPERATURE FROM COLOR INDEX OF Y OPHIUCHI

PHASE	MAG.			C.I.			C.I. TEMP.			AVE. C.I. TEMP.	BLACK-BODY TEMP.	MEAN TEMP. (2 DET.)
	λ 4200	λ 4500	λ 4800	λλ 4200–4500	λλ 4500–4800	λλ 4200–4500	λλ 4200–4500	λλ 4500–4800	λλ 4200–4800			
0d0	6.77	6.70	6.68	0.07	0.02	0.09	5500°	5900°	5700°	5700°	5500°	5600°
1.0	7.08	6.96	6.84	.12	.12	.24	5000	4550	4800	4780	4500	4650
2.0	7.36	7.19	7.09	.17	.10	.27	4600	4750	4700	4680	4250	4450
3.0	7.51	7.32	7.23	.19	.09	.28	4400	4900	4600	4630	4080	4350
4.0	7.67	7.42	7.33	.25	.09	.34	4000	4900	4350	4420	3960	4200
5.0	7.78	7.47	7.39	.31	.08	.39	3600	5000	4150	4250	3820	4000
6.0	7.85	7.53	7.40	.32	.13	.45	3600	4450	3950	3980	3680	3800
7.0	7.90	7.58	7.41	.32	.17	.49	3600	4150	3800	3850	3560	3700
8.0	7.93	7.60	7.42	.33	.18	.51	3550	4100	3750	3800	3500	3650
9.0	7.95	7.61	7.42	.34	.19	.53	3500	4000	3650	3720	3500	3600
10.0	7.94	7.60	7.41	.34	.19	.53	3500	4000	3650	3720	3520	3600
11.0	7.93	7.59	7.41	.34	.18	.52	3500	4100	3700	3770	3560	3700
12.0	7.92	7.59	7.40	.33	.19	.52	3550	4000	3700	3750	3640	3700
13.0	7.90	7.58	7.39	.32	.19	.51	3600	4000	3750	3780	3730	3800
14.0	7.84	7.56	7.39	.28	.17	.45	3800	4150	3950	3970	3850	3900
15.0	7.80	7.54	7.38	.26	.16	.42	3900	4250	4050	4070	3980	4000
16.0	7.67	7.42	7.29	.25	.13	.38	4000	4450	4200	4220	4300	4300
17.0	6.78	6.72	6.70	0.06	0.02	0.08	5600	5900	5750	5750	5490	5600

occurs for the variable at lower temperatures than for normal giants. This would necessitate a lower pressure in the stellar atmosphere. Saha's equation furnishes the logarithm of the ratio of the electronic pressures required to produce the same amount of ionization for two different temperatures,

$$\log \frac{P_1}{P_2} = 5048\, I \left(\frac{1}{T_2} - \frac{1}{T_1}\right) + 2.5 \log \frac{T_1}{T_2}.$$

Letting T_1 be the temperature from estimates of the spectral type (Table III), and T_2 the temperature from the distribution of inten-

[1] Ten Bruggencate, *Ann. Bosscha Obs.*, **2**, C23, 1928.

[2] Reesinck, Dissertation, Amsterdam, 1926.

[3] Ten Bruggencate, *op. cit.*, **2**, B58, 1927.

[4] Krieger, *op. cit.*, Table IV, cols. 3 and 13.

sity of the continuous background (Table VI), and taking 6 volts as an average ionization potential,[1] log (P_1/P_2) becomes 0.82 at phase $0^d.0$, and 2.7 at phase $10^d.0$. This indicates that the pressure of Y Ophiuchi (P_2) is about one-seventh of that of a normal giant (P_1) at phase $0^d.0$, and about one-five-hundredth at phase $10^d.0$.

It is interesting to note that the maximum radius computed from the observed magnitudes and temperatures by a method described by W. Baade[2] occurs near phase $9^d.0$–$10^d.0$, and minimum radius

TABLE VII

LINE-INTENSITY VARIATION OF Y OPHIUCHI, EXPRESSED IN TERMS OF THE INTENSITY OF THE CONTINUOUS BACKGROUND

Plate	Phase	$H\beta$	$H\gamma$	$H\delta$	Hydrogen Mean	G	H	K
274	$0^d.01$	0.384	0.402	0.372	0.386	0.328	0.736	0.776
253	1.12	.403	.475	.352	.410	.441	.820	.845
225	1.14	.363	.471	.319	.384	.326	.762	.874
209	2.41	.218	.341	.567	.375	.334	.....	.....
210	2.42	.384	.489	.....	.436	.277	.....	.....
215	4.38	.248	.362	.....	.305	.458	.517	.410
216	4.38	.341	.295	.415	.350	.500	.....	.....
232	8.36	.311	.226	.172	.236	.326	.....	.....
231	8.36	.296	.277	.385	.319	.288	.568	.414
241	10.24	.274	.279	.164	.242	.352	.....	.....
242	10.24	.291	.256	.253	.266	.365	.639	.539
246	12.23	.204	.284	.161	.216	.359	.669	.589
248	12.27	.323	.310	.212	.282	.383	.672	.587
268	14.12	.316	.380	.236	.311	.354	.699	.721
269	14.13	.286	.275	.226	.262	.419	.595	0.600
272	16.12	0.265	0.402	0.125	0.264	0.475	0.752	.....

near phase $0^d.0$. If the atmosphere were pulsating, maximum radius and minimum pressure, and further minimum radius and maximum pressure, would be expected to coincide, or nearly so.

The probable error of an individual determination of temperature and magnitude is $\pm 200°$ and $\pm 0.^m12$, respectively.

IV. LINE INTENSITIES

In the manner described by Yü,[3] the strengths of the β, γ, and δ lines of hydrogen, the H and K lines, and the G band were found in terms of the intensity of the continuous background (see Table VII and Fig. 3). The intensity variation of the hydrogen lines reflects

[1] B. Sticker, *Zeitschrift für Physik*, **61**, 562, 1930.

[2] *Astronomische Nachrichten*, **228**, 359, 1926.

[3] *Op. cit.*, **15**, 6, 1930.

the temperature variation as determined by various methods, with a maximum near phase $1^d.0$, and a minimum near $9^d.0$. The variation of the G band is not pronounced. The H and K lines reach maximum intensity near phase $1^d.0$ and minimum intensity near phase $6^d.0$.

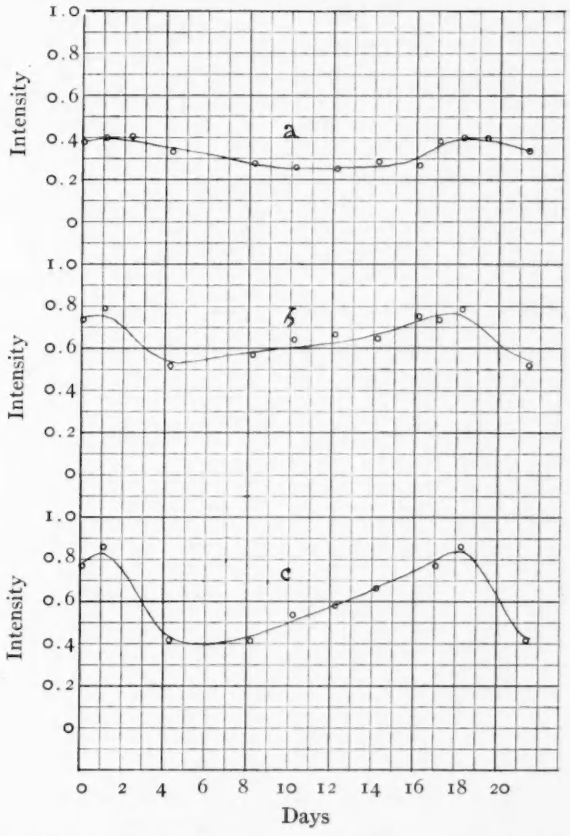


FIG. 3.—Variation of line intensities: (*a*) hydrogen; (*b*) H-line; (*c*) K-line

The writer is under obligation to the authorities of the Lick Observatory who placed at his disposal the instruments and material, and loaned to him the microphotometer tracings for detailed study.

ST. LOUIS UNIVERSITY
ST. LOUIS, MO.
August 1931

NOTE ON 27 CANIS MAJORIS

By OTTO STRUVE

ABSTRACT

The hypothesis advanced by McLaughlin is discussed. The helium lines show no visible emission and the star is altogether different from normal Be's. The displacements of the absorption lines are probably real and are caused by Doppler effect. It is possible that the lines are not produced in the reversing layer but in an extended shell of gas not directly connected with the star. The possibility that the changes in radial velocity are not caused by orbital motion has been emphasized in previous papers, but a decision must be postponed until more observations are available.

In a recent paper D. B. McLaughlin[1] has called attention to the similarity of 27 Canis Majoris to several other stars having bright lines of hydrogen. I quite agree with him that the observed changes in radial velocity may not be due to orbital motion—a possibility which I have emphasized in my articles on this star[2]—and the orbit which I have derived should be considered merely as a description of the velocity-curve. However, after a careful study of more than one hundred spectrograms I am not at all convinced that the shifts of the absorption lines can be attributed to the presence of bright lines. I discussed this matter in my paper, "Velocity-Curve and Spectral Variations of 27 Canis Majoris."[3]

McLaughlin suggests that what I measured as the principal component of 27 Canis Majoris is the central absorption of a variable emission line, and that the whole line—emission and central absorption—oscillates back and forth with a total range of about 220 km/sec. and with a period of approximately two thousand days. The reproductions on Plate IX show, however, no trace of emission in the helium lines. The absorption lines of helium are very strong and do not resemble the normal central absorptions of emission lines. They are, in fact, as strong as, or even stronger than, the normal helium lines in γ Pegasi.

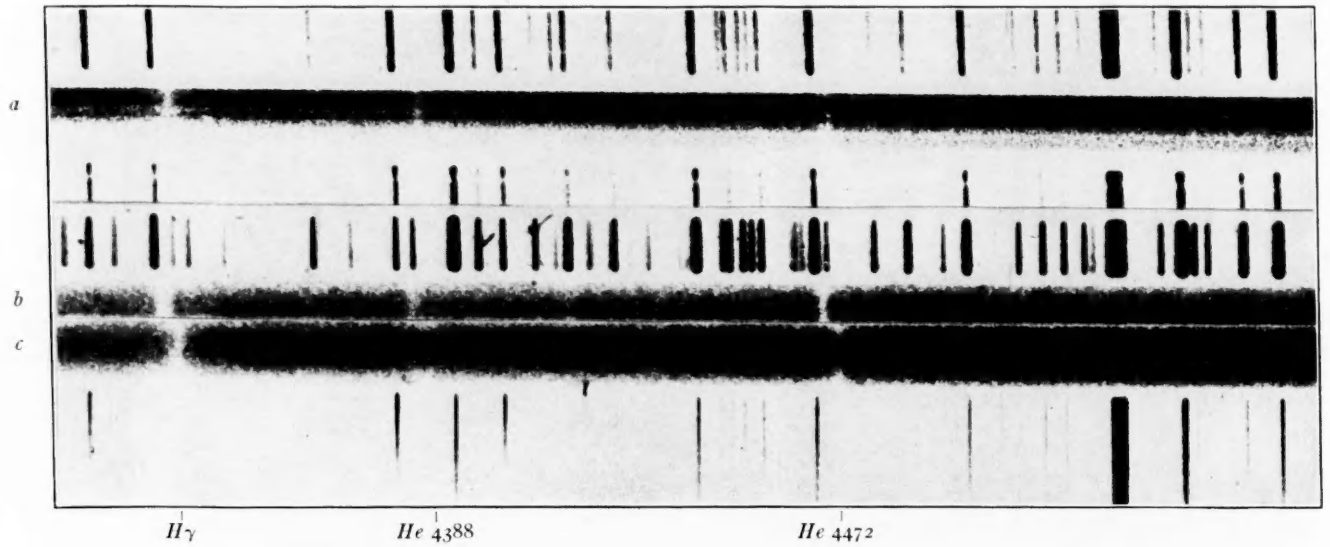
It should be remembered that while 27 Canis Majoris possesses variable bright lines of hydrogen, these are always faint, and cannot be seen on all plates.

[1] *Astrophysical Journal*, **75**, 60, 1932.

[2] *Astrophysical Journal* **65**, 285, 1927; *ibid.*, **68**, 115, 1928; *ibid.*, **73**, 307, 1931.

[3] *Ibid.*, **73**, 304–305, 1931.

PLATE IX



a) γ Pegasi (type B2)
b) 27 Can. Maj. 1928 Jan. 21; $V = -86$ km/sec. (Meas. vel. $= -84$; red to $\odot = -2$)
c) 27 Can. Maj. 1931 Jan. 16; $V = +119$ km/sec. (Meas. vel. $= +119$; red to $\odot = 0$)

There are many other stars which show much more conspicuous bright lines, but which fail to show an appreciable change in the radial velocity as determined from the central absorptions. Thus, the measures of C. D. Higgs[1] on π Aquarii show no correlation between the velocity of the central absorption of $H\beta$ and the marked changes in the relative intensities of the emission components.

I think it is fair to say, from an inspection of Plate IX, that the appearance of the spectrum itself suggests nothing disturbing (except the variation in the intensities of the absorption lines noted in my paper of June, 1931).[2] It is because of the statistical improbability of a mass of more than two thousand times that of the sun that we doubt the orbital interpretation.

I am inclined to think, with McLaughlin, that such a large mass is probably not physically real. In my discussion I found it difficult to reconcile all observations with one period. It is therefore possible that the period is not constant; and 27 Canis Majoris may resemble in this respect some of the other Be stars.

McLaughlin's[3] results for 25 Orionis and π Aquarii depend upon measurements of the central hydrogen absorptions. It is clear from the foregoing that the case of 27 Canis Majoris is altogether different.

It is practically certain that the shifts of the absorption lines are real, not apparent. It is also probable that they are caused by Doppler effect. The question is, *what* moves? Is it the star itself, with its reversing layer, or is it a more extended gaseous shell not directly connected with the star? The second alternative, which is somewhat related to the idea of McLaughlin, finds support in a class of peculiar stars which we have recently investigated at the Yerkes Observatory and which will be described elsewhere.[4] There are some indications that such extended gaseous shells may give rise to strong absorption lines, even when they do not produce noticeable emission. Whether or not this interpretation is acceptable in the case of 27 Canis Majoris remains to be seen.

[1] *Ibid.*, **72**, 187, 1930.

[2] *Ibid.*, **73**, 301, 1931.

[3] *Publications of the Observatory, University of Michigan*, **4**, 37, 1931.

[4] Struve and Swings, *Astrophysical Journal*, **75**, 1932.

Attention may be called to the fact that J. S. Plaskett's[1] massive star, B.D.+6°1309, has peculiar bright lines of *H* and of *He* II, and that another star of large mass, H.D.163181,[2] has bright lines of *H* and of *He* I.

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NOTE ADDED JANUARY 15.—A spectrogram taken 1931 Dec. 2.422 U.T. gives $V=+138$ km/sec. as derived from the lines of *H* and of *He* (*Hα*, +141 km/sec; *Hβ*, +143 km/sec; *He* 4388, +141 km/sec; *He* 4472, +126 km/sec.). I suspect that on the same plate *Mg* II 4481 is present as a faint diffuse line and that its displacement is less than that of *H* and of *He*.

[1] *Publications of the Dominion Astrophysical Observatory*, **2**, 147, 1922.

[2] M. L. Humason and S. B. Nicholson, *Astrophysical Journal*, **67**, 341, 1928.